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Analysis of the Effect of White Noise on the Halvorsen System of Variable-Order Fractional Derivatives Using a Novel Numerical Method

Mohammed Berir[1,2]
[1]Department of Mathematics, Faculty of Science, Al-Baha University, Al-Baha, Saudi Arabia.
[2]Department of mathematics, Faculty of Science, Bakht Al-Ruda University, Duwaym, Sudan.
E-mail: midriss@bu.edu.sa

Abstract

The Halvorsen System (HS) of Variable-Order (VO) Fractional Derivatives is a potent instrument for fractional differential equations and chaotic systems due to the influence of white noise. The study focuses on The Effect of White Noise on the Halvorsen System of (VO) Fractional Because the Halvorsen System is extensively utilized in nonlinear optics, fluid dynamics, and plasma physics, the chaos can be employed to investigate a wide array of relevant physical processes. To analyze the impact of noise we deduce that noise influences and stabilizes the chaotic Halvorsen system. We plotted numerous 3D and 2D graphical representations to explain how the white noise influences the chaotically systems. We applied these schemes to simulate the chaotic (VO) fractional Halvorsen system.

Keywords: *Impact of White Noise; Atangana-Baleanu; Halvorsen System; Variable-Order Fractional.*

1. Introduction

In recent decades, fractional differential and integral operators have attracted significant attention as expansions of their integer-order counterparts [1]. The sequence of these fractional operators can be of any arbitrary value. It is essential to recognize that fractional operators may be defined multiple times. [2] The Caputo sense is frequently regarded as one of the best recognized fractional derivatives. A major disadvantage of this operator is its solitary kernel, despite the presence of various beneficial attributes in this variant. Various definitions of fractional derivatives have been devised to address this issue. Examples of these definitions encompass the Caputo-Fabrizio (CF) [3] and Atangana-Baleanu (AB) [4] derivatives. The Caputo derivative is defined by employing a single kernel in lieu of the conventional (CF) formulation. Conversely, it has been employed in the definition of (AB). The new definitions, although addressing the issue of the singularity of the Caputo fractional derivative, also impose many limitations on their use. The characteristics of the kernel function render it exceedingly challenging to derive an in the (CF) fractional derivative. This is a consequence of the properties of the kernel function. Moreover, designing and implementing this form of fractional derivative is an exceptionally challenging endeavor. Nevertheless, although the notion presented by (AB)

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is user-friendly, to overcome the limitations outlined in the following section, we provide an innovative non-singular fractional differentiation formulation [5, 6].

Variable-order derivatives can simulate complicated dynamic behaviors that change over time, making them ideal for the Halvorsen system. This adaptability improves models of systems with non-constant differentiation orders, mirroring real-world events. The following sections will discuss variable-order derivatives and their types, such as Caputo and Atangana-Baleanu derivatives. Halvorsen System (VO) Derivatives Flexibility: Non-linear Halvorsen systems benefit from variable-order derivatives, which can adapt to system dynamics. Existence and Uniqueness: Research shows that variable-order Caputo-type fractional differential equations have unique solutions, making them useful in complicated systems [7]. Types of derivatives Caputo Derivative: This derivative allows fractional orders for functions that are not classically differentiable. It excels at modeling memory-effect systems. Atangana-Baleanu Derivative: This derivative generalizes fractional calculus with a non-local kernel. It helps capture past states' effects on current dynamics, which is important in Halvorsen's system. Variable-order derivatives have many benefits, but they also complicate computational methods and stability analysis, which must be considered in real implementations.

A valuable mathematical tool for a more in-depth analysis of objects that vary over time is the discipline of (VO) fractional calculus, which investigates integration and differentiation operators with (VO) fractional order [8]. This field of study is a subfield of fractional calculus. To be more specific, the mathematical systems that are described by this innovative approach exhibit higher levels of precision and sensitivity [9]. We would like to bring to your attention the fact that developing an analytical solution to issues applying (VO) fractional operators is frequently a very difficult task. Consequently, the advancement of numerical methods for solving these systems is a significant area of research. The Adams-B technique has long been recognized as an effective and powerful numerical technique for solving fractional systems. [10,11,12,13,14,15,16,17]. A constant-order numerical system was established by the authors not too long ago. This scheme is Lagrange polynomial. We make use of this approach to simulate (VO) fractional differential operators. to make the numerical methods described in [18, 19].

The study of chaotic systems that make use of variable-order fractional differential operators is extremely fascinating since these systems exhibit complicated dynamics and synchronization features. Classical fractional calculus is extended by these systems, which allows the order of differentiation to be changed; this means more flexibility and the occurrence of more complex behavior in chaotic systems. For a (VO) fractional derivative, which is a (VO) derivative of fractional order, the variable-order fractional derivatives can undergo constant modification influenced by time or other variables. Systems that become more dynamic in this way include the Liu system and chaotic systems. The addition of fractional derivatives of different orders allows them to behave in ways that are more complicated than they can in constant-order systems, such as being able to synchronize [20]. Synchronization in chaotic systems is achieved. It has been indicated that the active control methods can really realize synchronization in VO fractional chaotic systems, and the results of numerical simulation verify the correctness of theoretical analysis [21]. The synchronization of hyperchaotic systems with different fractional orders has been successfully implemented in our group, and this indicates possible applications in real life [22]. Applications and implications of the same the research in VO fractional systems is essential in particular with respect to the anomalous process modeling since it gives a more close approximation to the natural phenomena that may be complicated [23,24]. It would appear, based on the results, that the systems can exhibit high multistability and hidden attractors,a thing necessary when understanding chaotic dynamics. While the attention given to VO fractional systems illustrates that such systems can indeed display complex behavior and synchronization, one should not lose sight of the difficulties inherent in attempting to implement such systems physically and the need for reliable numerical methods to carry out accurate studies of their dynamics.

The main subject of this paper is the investigation of the effects of white noise on the Halvorsen system of (VO) fractional derivatives. This study illustrates the efficiency of white noise as a robust approach for the representation of nonlinear fractional differential equations and chaotic systems. Some chaotic models provided by 3D and 2D mappings are given in order to analyze its impact due to noise dynamic performances. These constitute an elaboration of previous studies, including those reported in reference [24]. The paper is organized as follows: In Section 2, include some basic definitions of (VO) fractional derivatives. In Section 3, we will analyze the effect that white noise has on the Halvorsen system of (VO) fractional derivatives. Further, the presence of white noise is considered with a new VO numerical scheme appearing in Section 4. Moreover, we will show that the presence of white noise affects chaotical systems using various graphical representations both in three dimensions and in two dimensions. in Section 5 Conclusion: the findings are summarized.

2. Basic Definition

We will employ these fundamental definitions of (VO) fractional derivatives in the following sections. This section provides these definitions.

Definition 2.1*The definition of (AB), as stated in reference [25], is delineated by the following instructions:*

$$ {}^{ABC}_{\ \ 0}D_t^{\beta(t)} = \frac{AB(\rho)}{1-\rho}\int_0^1 D_d(u)E_{\beta(t)}\left[\frac{\beta(t)}{1-\beta(t)}(1-u)^{\beta(t)}\right]du. \quad (1)$$

Furthermore, its integral is

$$ {}^{ABR}_{\ \ 0}I_f^{\rho}u(t) = \frac{1-\rho}{AB(\rho)}u(t)f(t) + \frac{\rho}{AB(\rho)\Gamma(\rho)}\int_0^t f(\psi)u(\psi)(t-\psi)^{\rho-1}d\psi. \quad (2)$$

Definition 2.2*([26]). The Caputo with a different order* $\beta(t)$*is given as.*

$$ {}^{C}_{0}D_t^{\beta(t)}\{Y(t)\} = \frac{1}{\Gamma\big(1-\beta(t)\big)}\int_0^t (t-\tau)^{-\beta(t)}Y'(\tau)\ d\tau. \quad (3)$$

Definition 2.3*([26]). White Noise in Fractional Calculus White noise in fractional calculus denotes a stochastic process within variable-order hyperchaotic systems. This noise is fundamentally defined:*

$$\mathcal{M}[\omega(\tau)\omega(\tau')] = \sigma^2\delta(\tau-\tau') \quad (4)$$

3. Halvorsen System Derivatives

3.1. Classical Halvorsen system

Within the framework of integer-order differential equations, the Halvorsen system, achaotic system like the Lorenz system, has been thoroughly investigated [27]. The following equations represent the Halvorsen system:

$$\frac{dX}{dt} = -AX - 4Y - 4Z - Y^2,$$
$$\frac{dY}{dt} = -AY - 4Z - 4X - Z^2, \tag{5}$$
$$\frac{dZ}{dt} = -\mathrm{AZ} - 4\mathrm{X} - 4\mathrm{Y} - \mathrm{X}^2.$$

3.2. The stochastic Halvorsen system of (VO) fractional derivatives

Finding a chaotic Halvorsen system of (VO) fractional derivatives is a very difficult task in many real settings. This is because the existence of both nonlinearity and Stochasticity makes the task extremely difficult. Researchers, on the other hand, have utilized a variety of methods, such as (VO) fractional without for the stochastic we take into consideration the stochastic Halvorsen system of variable-order fractional in this working paper:

$${}^{ABC}_{\ 0}D_t^{\beta(t)} X(t) = -AX - 4Y - 4Z - Y^2 + \sigma_1 H_1(t) dB(t),$$
$${}^{ABC}_{\ 0}D_t^{\beta(t)} Y(t) = -AY - 4Z - 4X - Z^2 + \sigma_2 H_2(t) dB(t), \tag{6}$$
$${}^{ABC}_{\ 0}D_t^{\beta(t)} Z(t) = -AZ - 4X - 4Y - X^2 + \sigma_3 H_3(t) dB(t).$$

here ${}^{ABC}_{\ 0}D_t^{\beta(t)}$ is (AB) sense with variable-order, $\beta(t) = \tanh(t+1)$ is variable-order fractional, $B(t)$ describes is the white noise (Gaussian process) and $\sigma(i)$ for, $i = 1; 2; 3$ denote the intensity of noise or the intensity of the stochastic environment. All the variables X, Y, Z in the above models are non-negative.

4. The Stochastic Halvorsen System of (VO) Fractional Atangana Derivative using A Numerical Scheme

This analysis presents a numerical method for a fractional system that incorporates stochastic elements. The Atangana fractional derivative utilizes a variable-order fractional approach. Utilizing the fundamental theorem of fractional calculus, we obtain:

$$ {}_0^{ABC}\mathcal{D}_t^{\beta(t)}Y(t) = f(t, y(t)) + \sigma Y(t)dB(t) \tag{7} $$

$$ \begin{aligned} Y(t) - Y(0) &= \frac{1-\beta(t)}{B(\alpha(t))} f(t, Y(t)) \\ &+ \frac{\beta(t)}{\Gamma(\beta(t))B(\alpha(t))} \int_0^t f(\theta, Y(\theta))(t-\theta)^{\alpha(t)-1} d\theta \\ &+ \int_0^t \sigma y(t) dB(t) \end{aligned} \tag{8} $$

Where $ABC(\beta(t)) = 1 - \beta(t) + \frac{\beta(t)}{\Gamma(\beta(t))}$ is a normalization function.
At t_{n+1}, we have:

$$ \begin{aligned} Y(t_{n+1}) - Y(0) = &\frac{\Gamma(\beta(t))(1-\beta(t))}{\Gamma(\beta(t))(1-\beta(t)) + \beta(t)} f\big(t_n, Y(t_n)\big) \\ &+ \frac{\beta(t)}{\Gamma(\beta(t)) + \beta(t)(1-\Gamma(\beta(t)))} \sum_{m=0}^{n} \int_{t_m}^{t_{m+1}} f(\theta, Y(\theta))(t_{n+1}-\theta)^{\beta(t)-1} d\theta \\ &+ \int_{t_m}^{t_{m+1}} \sigma Y(t) dB(t). \end{aligned} \tag{9} $$

Using the Lagrange interpolation, the function $f(\tau, Y(\tau))$ is approximated by:

$$ P_k(\theta) \simeq \frac{f(t_m, Y_m)}{h}(\theta - t_{m-1}) - \frac{f(t_{m-1}, Y_{m-1})}{h}(\theta - t_m), \tag{10} $$

where Eq. (10) is replaced in Eq. (9) to obtain:

$$ \begin{aligned} Y_{n+1}(t) = &y_0 + \frac{\Gamma(\beta(t))(1-\beta(t))}{\Gamma(\beta(t)(t))(1-\beta(t)(t)) + \beta(t)(t)} f\big(t_n, Y(t_n)\big) \\ &+ \frac{\beta(t)}{\Gamma(\beta(t)(t)) + \beta(t)(t)(1-\Gamma(\beta(t)(t)))} \sum_{m=0}^{n} \\ &\times \Bigg(\frac{f(t_m, Y_m)}{h} \int_{t_m}^{t_{m+1}} (\theta - t_{m-1})(t_{n+1}-\theta)^{\beta(t)-1} d\theta \\ &- \frac{f(t_{m-1}, y_{m-1})}{h} \int_{t_m}^{t_{m+1}} (\theta - t_m)(t_{n+1}-\theta)^{\beta(t)-1} d\theta \Bigg) \\ &+ \int_{t_m}^{t_{m+1}} \sigma y(t) dB(t). \end{aligned} \tag{11} $$

Integrating (9) and replacing in (11), the following approximation is obtained

$$Y_{n+1}(t) = Y_0 + \frac{\Gamma(\beta(t))(1-\beta(t))}{\Gamma(\beta(t))(1-\beta(t)) + \beta(t)} f\big(t_n, Y(t_n)\big)$$
$$+ \frac{1}{(\beta(t)+1)((1-\beta(t))\Gamma(\beta(t)) + \beta(t)} \sum_{m=0}^{n}$$
$$\times \big(h^{\beta(t)} f(t_m, Y_m)\big((n+1-m)^{\beta(t)}(n-m+2+\beta(t)) \tag{12}$$
$$- (n-m)^{\beta(t)}(n-m+2+2\beta(t))\big)$$
$$- h^{\beta(t)} f(t_{m-1}, Y_{m-1})\big((n+1-m)^{\beta(t)+1} - (n-m)^{\beta(t)}$$
$$\times (n-m+1+\beta(t))))$$
$$+\beta(t)\sigma y(t)(c_n)\big(B(t_{n+1}) - B(t_n)\big)$$

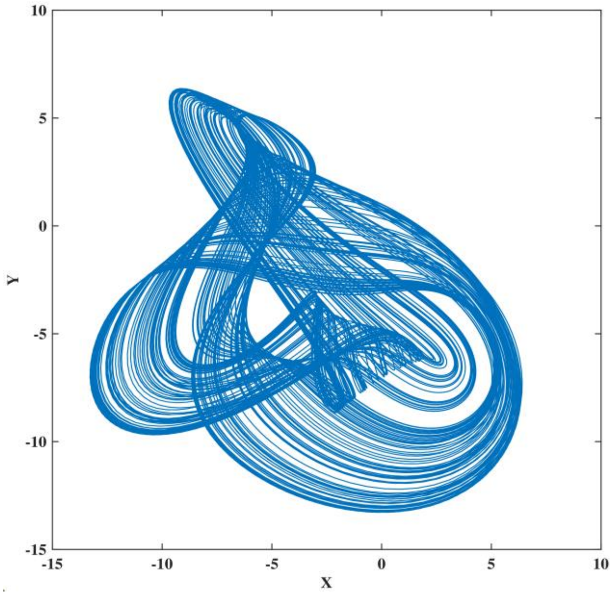
Finally, we obtain the following numerical representation of the system (6):

$$X_{n+1}(t) = X_0 + \frac{\Gamma(\beta(t))(1-\beta(t))}{\Gamma(\beta(t))(1-\beta(t)) + \beta(t)} f\big(t_n, X(t_n)\big)$$
$$+ \frac{1}{(\beta(t)+1)((1-\beta(t))\Gamma(\beta(t)) + \beta(t)} \sum_{m=0}^{n}$$
$$\times \big(h^{\beta(t)} f(t_m, X_m)\big((n+1-m)^{\beta(t)}(n-m+2+\beta(t)) \tag{13}$$
$$- (n-m)^{\beta(t)}(n-m+2+2\beta(t))\big)$$
$$- h^{\beta(t)} f(t_{m-1}, X_{m-1})\big((n+1-m)^{\beta(t)+1} - (n-m)^{\beta(t)}$$
$$\times (n-m+1+\beta(t))))$$
$$+\beta(t)\sigma y(t)(c_n)\big(B(t_{n+1}) - B(t_n)\big)$$

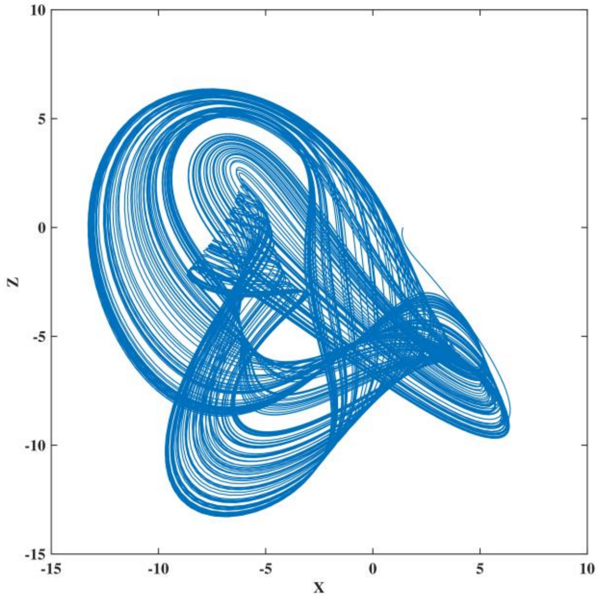
$$Z_{n+1}(t) = Z_0 + \frac{\Gamma(\beta(t))(1-\beta(t))}{\Gamma(\beta(t))(1-\beta(t)) + \beta(t)} f\big(t_n, Z(t_n)\big)$$
$$+ \frac{1}{(\beta(t)+1)((1-\beta(t))\Gamma(\beta(t)) + \beta(t)} \sum_{m=0}^{n}$$
$$\times \big(h^{\beta(t)} f(t_m, Z_m)\big((n+1-m)^{\beta(t)}(n-m+2+\beta(t)) \tag{14}$$
$$- (n-m)^{\beta(t)}(n-m+2+2\beta(t))\big)$$
$$- h^{\beta(t)} f(t_{m-1}, Z_{m-1})\big((n+1-m)^{\beta(t)+1} - (n-m)^{\beta(t)}$$
$$\times (n-m+1+\beta(t))))$$
$$+\beta(t)\sigma y(t)(c_n)\big(B(t_{n+1}) - B(t_n)\big)$$

5. The Impact of White Noise on the Halvorsen System

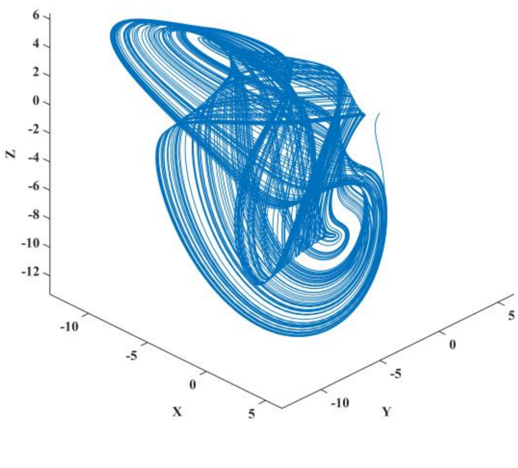
This section covers the (VO) fractional derivatives system of Halvorsen. A novel numerical method investigates the system with and without white noise. We numerically investigate the effect of white noise on the variable orders of chaotic dynamics through simulations. As seen in Figure 1(A-F), The Halvorsen System of Variable-Order Fractional Derivatives When the noise is absent (i.e., σ = 0), there are several types of simulations of the system (6). the following phase portrait behavior: (A) X-Y plane phase portrait at σ = 0; (B) X-Z plane phase portrait at σ = 0; (C) X-Y-Z plane phase portrait at σ = 0; (D) X-time plane time series at σ = 0; (E) Y-time plane time at σ = 0; (F) Z-time plane time at σ = 0. Figure 2(A-F), The Halvorsen System of Variable-Order Fractional Derivatives When the noise is present (i.e., σ = 1), there are several types of simulations of the system (6). the following phase portrait behavior: (A) X-Y plane phase portrait at σ = 1; (B) X-Z plane phase portrait at σ = 1; (C) X-Y-Z plane phase portrait at σ = 1; (d) X-time plane time series at σ = 1; (E) Y-time plane time at σ = 1; (F) Z-time plane time at σ = 1. As Figure 3(A-F) illustrates, there are more effective types of white noise when the noise is strong. However, noise adversely affects the system, and the chaotic behavior flattens as the noise level increases. finally Figure 4(A-F). Comparison between the time series the system behaves differently when the noise is absent or present.



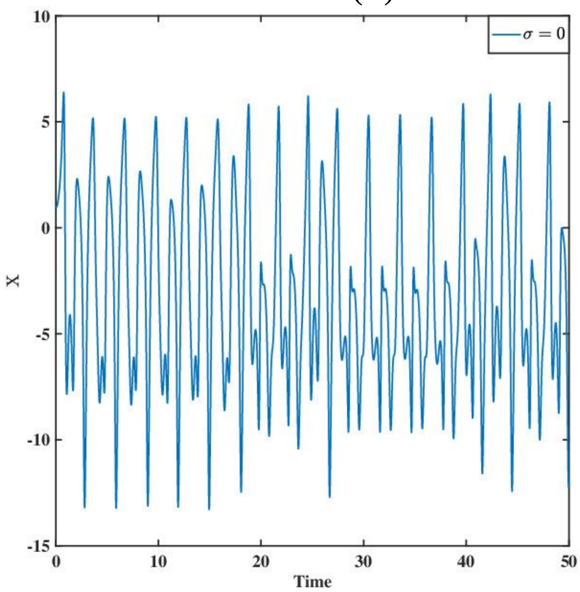
(A)



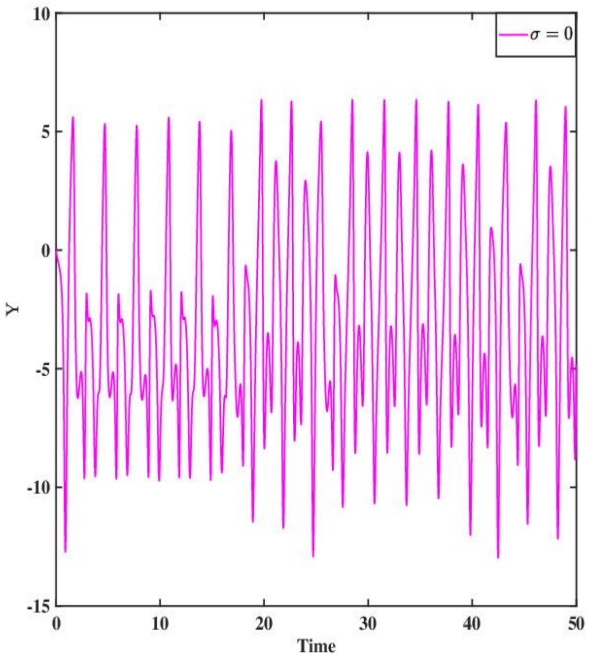
(B)



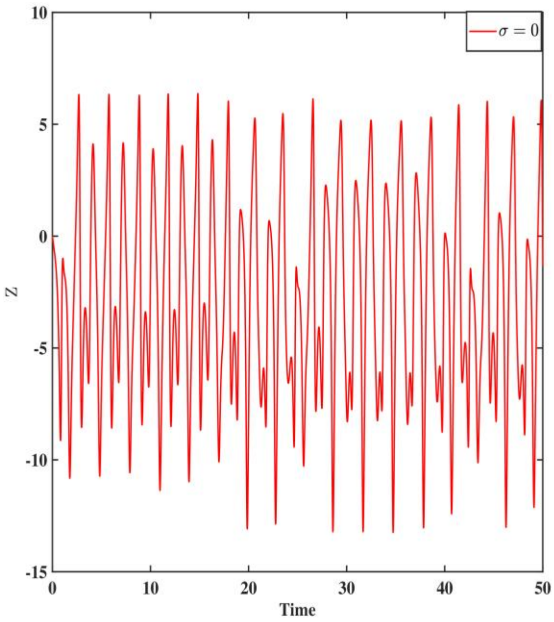
(C)



(D)

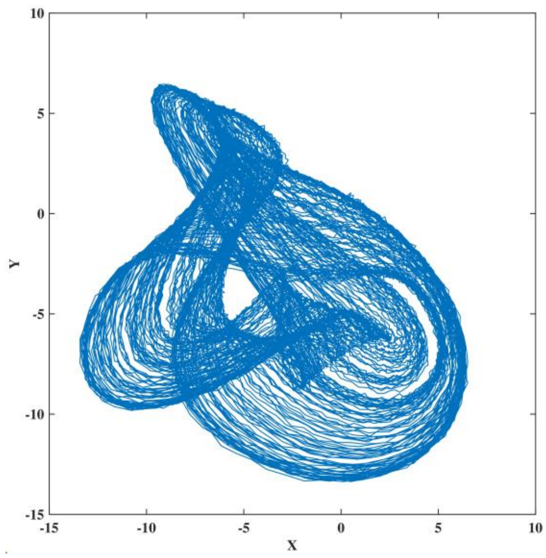


(E)

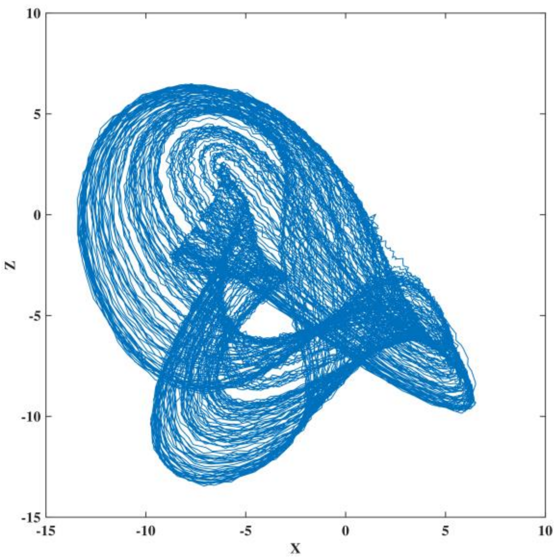


(F)

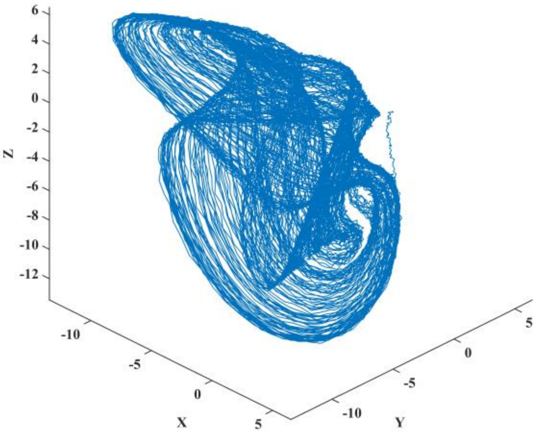
Figure 1. (A–F) shows a 3D and a 2D profile when there is no noise present (i.e., $\sigma = 0$) for the system (6).



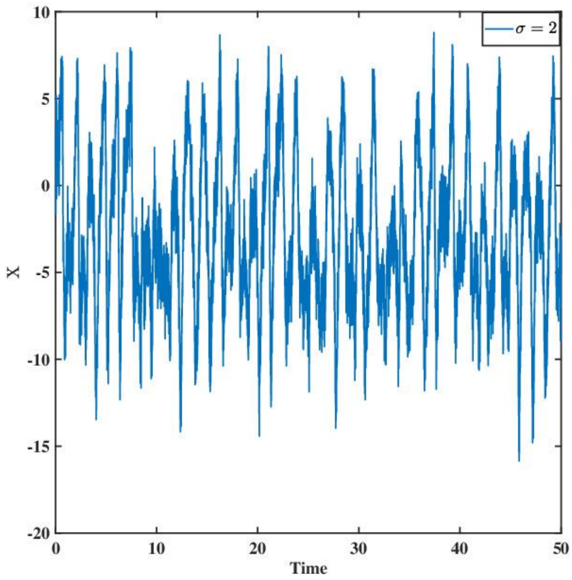
(A)



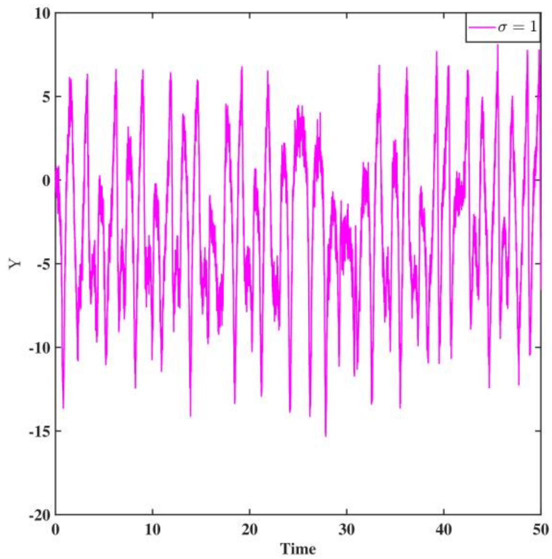
(B)



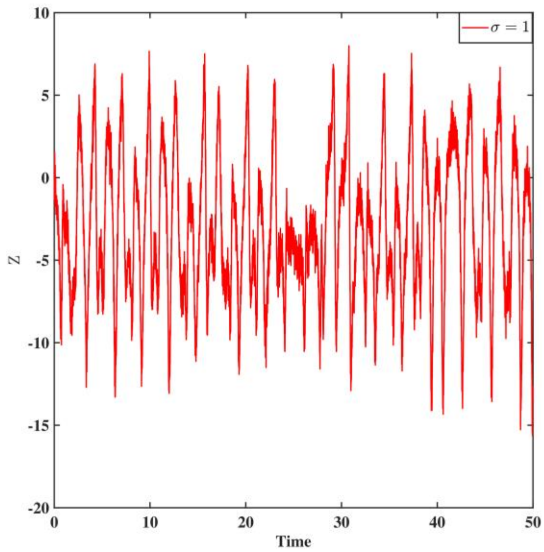
(C)



(D)



(E)



(F)

Figure 2 (A–F) illustrates a 3D and a 2D profile for the system (6) when there is noise present. (i.e., $\sigma = 1$).

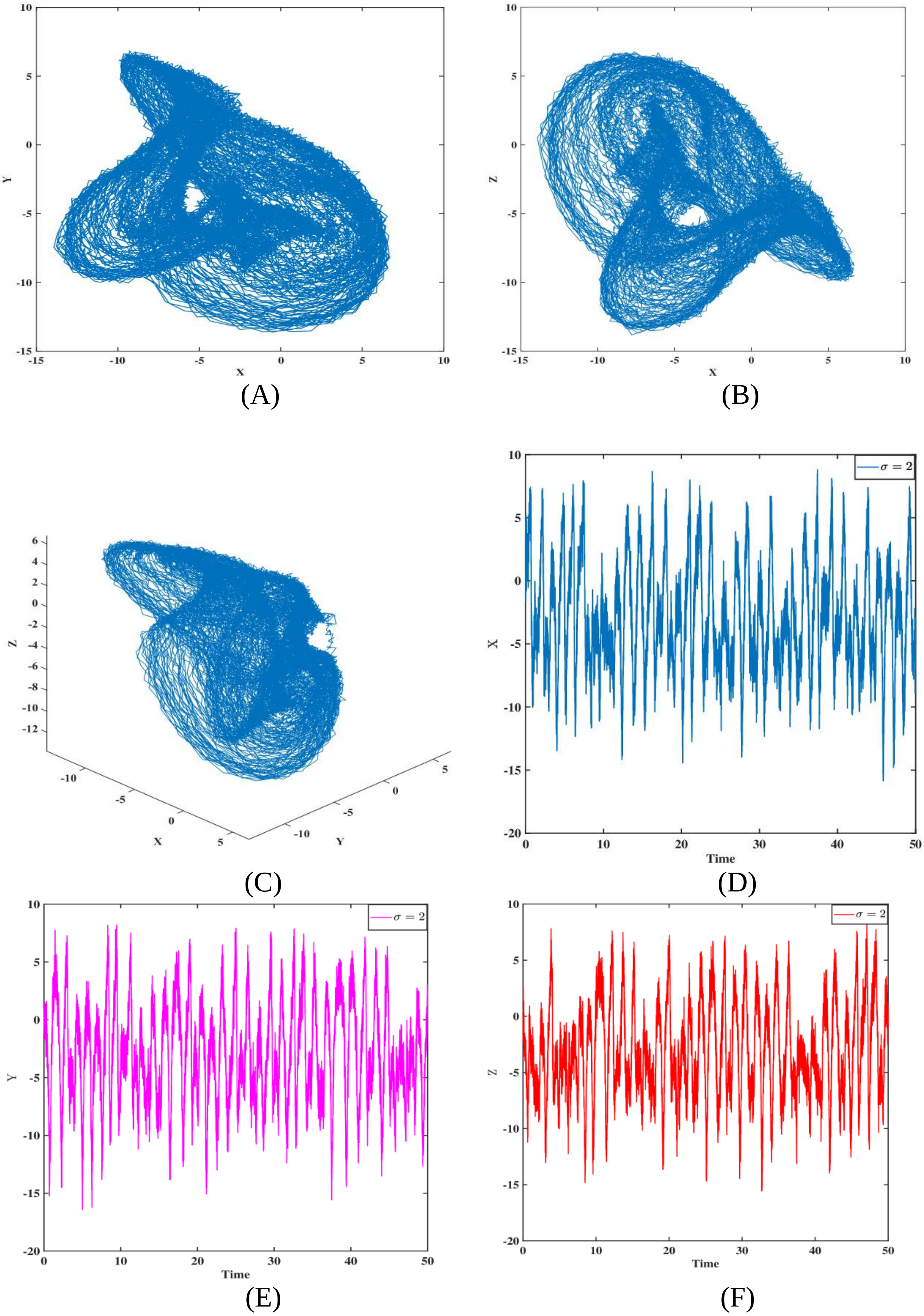
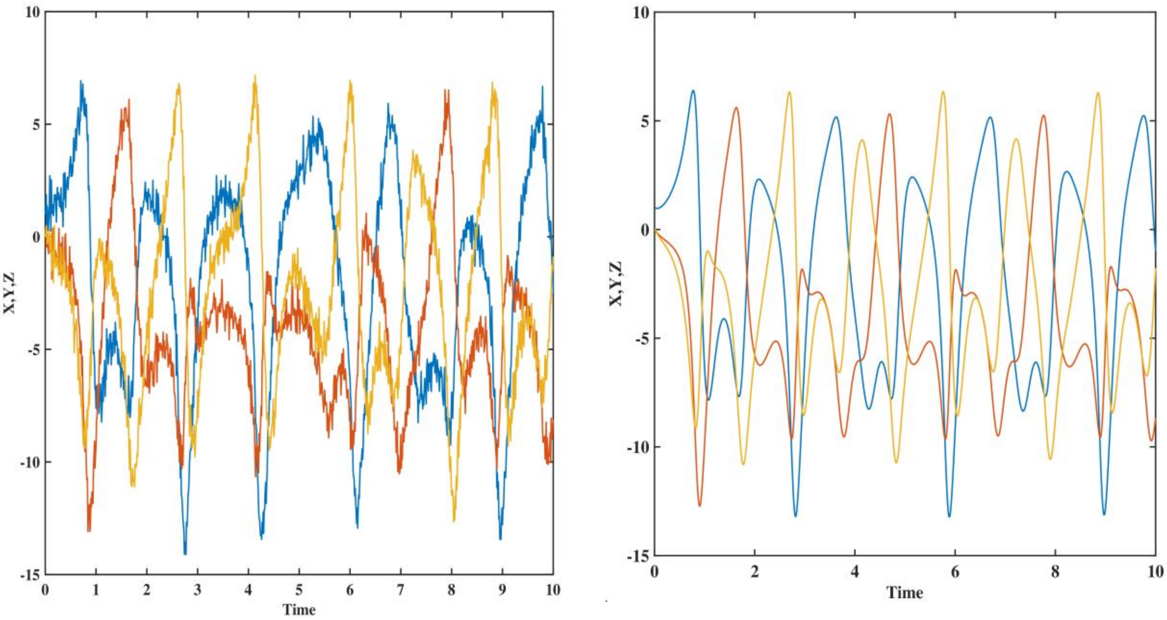


Figure 3 (A–F) illustrates a 3D and a 2D profile for the system (6) when there is noise present. (i.e., $\sigma = 2$).



(A) (B)

Figure 4: (A–B) Comparison between the time series When the noise is absent and present at $\sigma = 0.5 \sigma = 0$.

6. Conclusion

In summary, (VO) differential operators are a useful tool for simulating stochastic fractional differential equations. This research illustrates that white noise substantially affects the dynamics of the Halvorsen system. This paradigm enhances the stability of modeling nonlinear fractional differential equations and chaotic systems. The simulations are presented in Section 4, along with numerical solutions for the Halvorsen system. The chaotic behavior of the system (ABC) fractional is significantly influenced by white noise, as illustrated in Figures 2–3. This has allowed us to highlight the impact of multiplicative noise on the chaotic simulation of the system and the stability of those solutions. The utilized numerical approaches have demonstrated good outcomes. This study improves the understanding of the intricate interplay between chaos and noise in complex systems. In the future, we may analyze disease systems using a novel analytical method that includes additive noise [28, 29, 30, 31, 32].

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