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Improved Genetic Algorithm to Solve Asymmetric Traveling Salesman Problem

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Abstract

The asymmetric traveling salesman problem (ATSP) is a combinatorial problem of great importance where the cost matrix is not symmetric, which complicates its resolution. The genetic algorithms (GAs) are a meta-heuristics methods used to solve transportation problems that have proved their effectiveness to obtain good results. However, improvements can be made by adapting the crossover operator as a primordial operator in GAs. In this work, we propose an adapted XIM crossover operator for the ATSP in order to improve the optimal solution obtained by GAs. Numerical simulations are performed and discussed for different series of standard instances showing the improvement of the optimal solution by the proposed genetic operator.

Keywords: *Combinatorial problem; ATSP; Genetic algorithm; crossover operator*

1 Introduction

The asymmetric traveling salesman problem (ATSP) is a combinatorial problem of great importance which can model several real problems, especially in distribution, and in vehicle routing problems. This importance explains the large number of works which studies the different aspects of this problem and several survey that have been written about it in the past few decades which present different formulations for the ATSP such as [1], and present the different exact, heuristic and metaheuristic procedures for the ATSP [2, 3, 4, 5]. The ATSP is an extension of the traveling salesman problem (TSP), and it can be stated as follows: A traveler must visit a finite number of cities passing by each of them once and return to the starting city. The only difference between these two problems is that in the ATSP, the distance from a city "A" to the city "B" is not equal, as in the TSP, to that between the city "B" to the city "A" [6].

The ATSP problem is classified as an NP-complete problem such as TSP [7]. There are some intuitive methods to find the approximate solutions [8, 9], but all of these methods have exponential complexity, they take too much computing time or require too much memory. Then, meta-heuristics methods should be exploited.

The importance of ATSP is that many of the real-world problem such as single vehicle routing problem and different hard multivehicle routing problems, that cannot be modeled as an ATSP, can be solved by an polynomial transformation into an ATSP through an asymmetric generalized traveling salesman problem (AGTSP) by the same algorithms developed for the ATSP without any modification [10, 11, 12].

An approximation algorithm, like the Genetic Algorithms, Ant Colony [13] and Tabu Search [7, 14], is a way of dealing with NP-completeness for optimization problem. This technique does not guarantee the best solution but it is to come as close as possible to the optimum value in a reasonable amount of time which is at most polynomial time.

The genetic algorithm is a one of the family of evolutionary algorithms [15]. The population of a genetic algorithm (GA) evolves by using genetic operators inspired by the evolutionary in biology, "The survival is the individual most suitable to the environment". Darwin discovered that species evolution is based on two components: the selection and reproduction. The selection provides a reproduction of the strongest and more robust individuals, while the reproduction is a phase in which the evolution run. These algorithms were modeled on the natural evolution of species. We add to this evolution the concepts of observed properties of genetics (Selection, Crossover, Mutation, etc), from which the name Genetic Algorithm. They attracted the interest of many researchers, starting with Holland [16], who developed the basic principles of genetic algorithm, and Goldberg [17] that has used these principles to solve specific optimization problems. Other researchers have followed this path [18, 19].

In a genetic algorithm, a population of individuals (possible solution) is randomly selected. These individuals are subject to several operations, called genetic operators (selection, crossover, mutation, insertion, ..) to produce a new population containing in principle better individual. This population evolves more and more until a stopping criterion is satisfied and declaring obtaining optimal best solution. Thus; the performance of a genetic algorithm depends on the choice of operators who will intervene in the production of the new populations [20, 21]. Among the difficulties in GAs is the parameter setting and the choice of the crossover operator adapted to the problem which is the most influent operator in the GAs.

In this work, we consider the resolution of the ATSP by genetic Algorithms where we will present each individual by the most adapted method of data representation, the path representation method, which is the most natural representation of a tour (a tour is encoded by an array of integers representing the successor and predecessor of each city). We attempt to develop a new crossover operator in order to find the best meta-heuristic solution.

This paper is organized as follows. The mathematical formulation of the ATSP is presented in Section 2. In order to enhance the optimization performance, the standard GA and the developed adapting crossover operator to improve the evolution of the GA for the ATSP are established in Section 3. In Section 4, numerical simulations were performed through many ATSPs standard instances. The comparison with previous work shows that the proposed XIM crossover operator for ATSP improves the performance of a genetic algorithm and provides better solutions.

2 Mathematical formulation for ATSP

The Traveling Salesman Problem (TSP) is one of the most intensively studied problems in computational mathematics. The Asymmetric Traveling Salesman Problem (ATSP) as extension of (TSP) has the same importance which is an NP-hard problem in combinatorial optimization studied in operations research and theoretical computer science.

In a practical form, the problem is that a traveling salesman must visit every city in his territory exactly once and then return to the starting point. Given the cost of travel between all cities that is not symmetric, how should he plan his itinerary for minimum total cost of the entire tour?

The search space for the ATSP is a set of permutations of n cities and the optimal solution is a permutation which yields the minimum cost of the tour.

In other words, an ATSP of size n is defined by:

We consider a set of points $v = \{v_1, v_2, \ldots, v_n\}$ which v_i is a city.

The mathematical formulation of the ATSP is given by:

$$Min\,\{f(T), T = (T[1], T[2], \ldots\ldots, T[n], T[1])\} \quad (1)$$

where;
f is the evaluation function calculates the adaptation of each solution of the problem given by the following formula:

$$f = \sum_{i=1}^{i=n-1} d(T(i), T(i+1)) + d(T(n), T(1)) \tag{2}$$

with *d* define a non symmetric distance matrix and *d(i, j)* the distance between the city v_i and v_j such that:

$$d(i, j) \neq d(j, i) \tag{3}$$

$T[i]$ is a permutation on the set *{1, 2, . . . ,n}*.
T = (T[1], T[2], ... , T[n], T[1]) is the scheduling form of a solution of the ATSP.
n is the number of cities.

3 Adapted genetic Algorithm for ATSP

3.1 Principle of genetic algorithms

The genetic algorithm is a one of the family of evolutionary algorithms. The population of a genetic algorithm (GA) evolves by using genetic operators inspired by the evolutionary in biology. These algorithms were modeled on the natural evolution of species. We add to this evolution concepts the observed properties of genetics (Selection, Crossover, Mutation, etc), from which the name Genetic Algorithm. They attracted the interest of many researchers, starting with Holland [16], who developed the basic principles of genetic algorithm, and Goldberg [17] who has used these principles to solve specific optimization problems.

Irrespective of the problems treated, genetic algorithms are based on six principles [20]:

- Each treated problem has a specific way to encode the individuals of the genetic population. A chromosome (a particular solution) has different ways of being coded: numeric, symbolic, matrix or alphanumeric;
- Creation of an initial population formed by a finite number of solutions;
- Definition of an evaluation function (fitness) to evaluate a solution;
- Selection mechanism to generate new solutions, used to identify individuals in a population, there are several methods in the literature, citing the method of selection by rank, roulette, by tournament, random selection, etc.;
- Reproduce the new individuals by using Genetic operators:
    1. Crossover operator: it is a genetic operator that combines two chromosomes (parents) to produce a new chromosome (children) with crossover probability P_c ;

2. Mutation operator: it avoids establishing a uniform population unable to evolve. This operator used to modify the genes of a chromosome selected with a mutation probability P_m;

- Insertion mechanism: to decide who should stay and who should disappear.
- Stopping test: to make sure about the optimality of the solution obtained by the genetic algorithm.

3.2 Adapted crossover operator for ATSP

In a genetic algorithm, selected individuals are subjected to a series of operations through "genetic operators". The crossover operator is the one that most influences in the reproduction of new individuals with the hope that it creates a better offspring [20].

There are several crossover operators [21, 22], citing those using a crossover mask (**Fig 1.**), which is a vector generated randomly by identical length parents channels and composed of 0 and 1. When the bit mask is 0, the child inherits the bit of the first parent; otherwise it inherits that of the second parent. The second child is the complement of the first. This crossing can be regarded as a generalization of multi-function without prior knowledge of the crossing point.

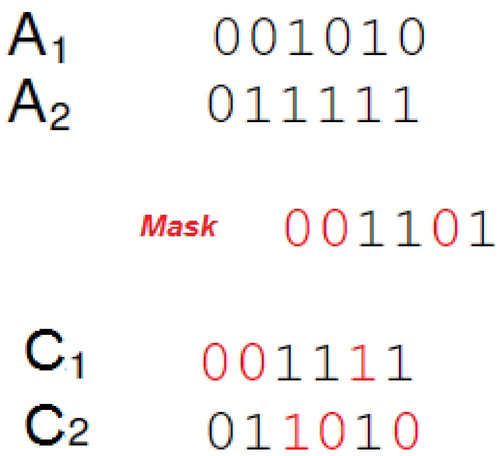


Fig. 1 Representation of a crossover mask.

In this paper, we present Crossover Inverse Mask operator (XIM) as a new operator which is based on basic principle of reversing the crossover mask. Given two parent chromosomes, two random crossover points are selected partitioning them into a left, middle and right portion. The ordered two-point crossover behaves in the following way: child1 inherits its left and right section from parent1, and its middle section is determined and finally the holes are created at the retranscription of the genotype, if $x_{ji} \in \{x_{k,a}, \ldots\ldots x_{k,b}\}$ then $x_{j,i}$ is a hole. We explain the mechanism of the Crossover Inverse Mask (XIM) using in the example (**Fig. 2**). The algorithm (**Fig. 3**) shows the crossover method XIM.

Parent 1	1	16	8	13	3	5	9	7	2	14	11	10	17	4	12	6	15	1
Parent 2	1	13	16	5	4	8	12	9	2	7	15	14	17	11	10	3	6	1
Crossover Mask	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Inverse Mask	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
IM X P2			10	11	17	14	15	7	2	9	12	8	4	5	16			
Dep + Arr	1	-	10	11	17	14	15	7	2	9	12	8	4	5	16	-	-	1
Miss Elemnts in P1	1	~~16~~	10	11	17	14	15	7	2	9	12	8	4	5	16	6	~~15~~	1
Missing Ei є P1								13	3									
Insert in C1	1	13	10	11	17	14	15	7	2	9	12	8	4	5	16	6	3	1
Child 1	1	13	10	11	17	14	15	7	2	9	12	8	4	5	16	6	3	1
Child 2	1	16	12	4	17	10	11	14	2	7	9	5	3	13	8	15	6	1

Fig. 2 Example of XIM operator

Regarding other genetic operators used in the implementation of this approach, we have chosen as a selection method the most common type – Roulette [21], which the individuals are given a probability P_i of being selected (4) that is directly proportionate to their fitness.

$$\frac{1}{N-1} \times \left(1 - \frac{f_i}{\sum_{j \in Population} f_j}\right) \tag{4}$$

To enhance the genetic reproduction, we integrated the operator Twors Mutation [20] as mutation operator with the mutation probability P_m between 0.0045 and 0.117 it depends on the ATSP instances deployed.

We used the method of inserting elitism that consists in copy the best chromosome from the old to the new population. This is supplemented by the solutions resulting from operations of crossover and mutation, in ensuring that the population size remains fixed from one generation to another [23].

Input: **Parents** $x_1 = [x_{1,1}, x_{1,2}, \dots\dots, x_{1,n}]$ and $x_1 = [x_{2,1}, x_{2,2}, \dots\dots, x_{2,n}]$
Output: **Children** $y_1 = [y_{1,1}, y_{1,2}, \dots\dots, y_{1,n}]$ and $y_2 = [y_{2,1}, y_{2,2}, \dots\dots, y_{2,n}]$

Initialize

- Initialize y_1 and y_2 being a empty genotypes;
- Choose two crossover points a and b such that $1 \le a \le b \le n$;

$j_1 = j_2 = k = b+1$;

- Generating a crossover mask defined by the two points a and b

Repeat
    $j \leftarrow$ Index where we find $x_{2,i}$, in X_1;
    $y_{1,j} = x_{1,j}$;
    $y_{2,j} = x_{2,j}$;
    $i = j$;
Until $x_{2,i} \notin y_1$
reverse crossover mask
for each i between a and n **do**
    if $x_{1,i} \notin \{x_{2,a}, \dots, x_{2,b}\}$ **then** $y_1 = [y_1\ x_{1,i}]$;
    if $x_{2,i} \notin \{x_{1,a}, \dots, x_{1,b}\}$ **then** $y_2 = [y_2\ x_{2,i}]$;
endfor
$i = 1$;
Repeat
    if $x_{1,i} \notin \{x_{2,a}, \dots, x_{2,b}\}$ ***then*** $y_{1,j1} = x_{1,k}$; j_1++;
    if $x_{2,i} \notin \{x_{1,a}, \dots, x_{1,b}\}$ ***then*** $y_{2,j1} = x_{2,k}$; j_2++;
    k=k+1;
Until $i \le n$
For each gene not yet initialized **do**
    $\mathbf{y_{1,i} = x_{2,i}}$;
    $\mathbf{y_{2,i} = x_{1,i}}$;
Endfor
$y_1 = [y_{1,1} \dots\dots y_{1,a-1}\ x_{2,a} \dots\dots x_{2,b}\ y_{1,a} \dots\dots y_{1,n-a}]$;
$y_2 = [y_{2,1} \dots\dots y_{2,a-1}\ x_{1,a} \dots\dots x_{1,b}\ y_{2,a} \dots\dots y_{2,n-a}]$;

Fig. 3 Algorithm of Crossover operator XIM

4 Numerical result

In this study, FULL-MATRIX [2] br17, ft53, ft70, ftv33, ftb35, ftb38, ftv44, ftv47, ftv55, ftv64, ftv70, ftv170, kro124p, p43, rbg323, rbg358, rbg433, and ry48p are used from Standard asymmetric TSP libraries [24] to show the effectiveness of the XIM method proposed in this paper with a crossover probability Pc varies according to the used crossover mask. All programs in the C++ language were developed for the tests of proposed operator and to compare it with the GA presented in the work [25]. The performance tests were performed with a laptop with Intel Corei31.7 GHz, 4GB RAM. There are 33 cities in the FTV33, 44 cities in the FTV44, and 64 cities in the FTV64 library. Performance tests were executed for GAM and XIM.

Depending on the change in the global fitness values of the XIM and GAM in each iteration are presented in Figure 4. Also, depending on the iteration, the global minimum change for FTV33 is shown graphically in Fig. 4(a), FTV44 in Fig. 4(b) and FTV64 are shown in Fig. 4(c). The test results show that the values obtained from the XIM operator mostly converge to the optimal solution.

Moreover, it can be seen that the proposed operator XIM always reaches better global minimum rates in general for all ATSP series (Fig. 5, 6 and 7).

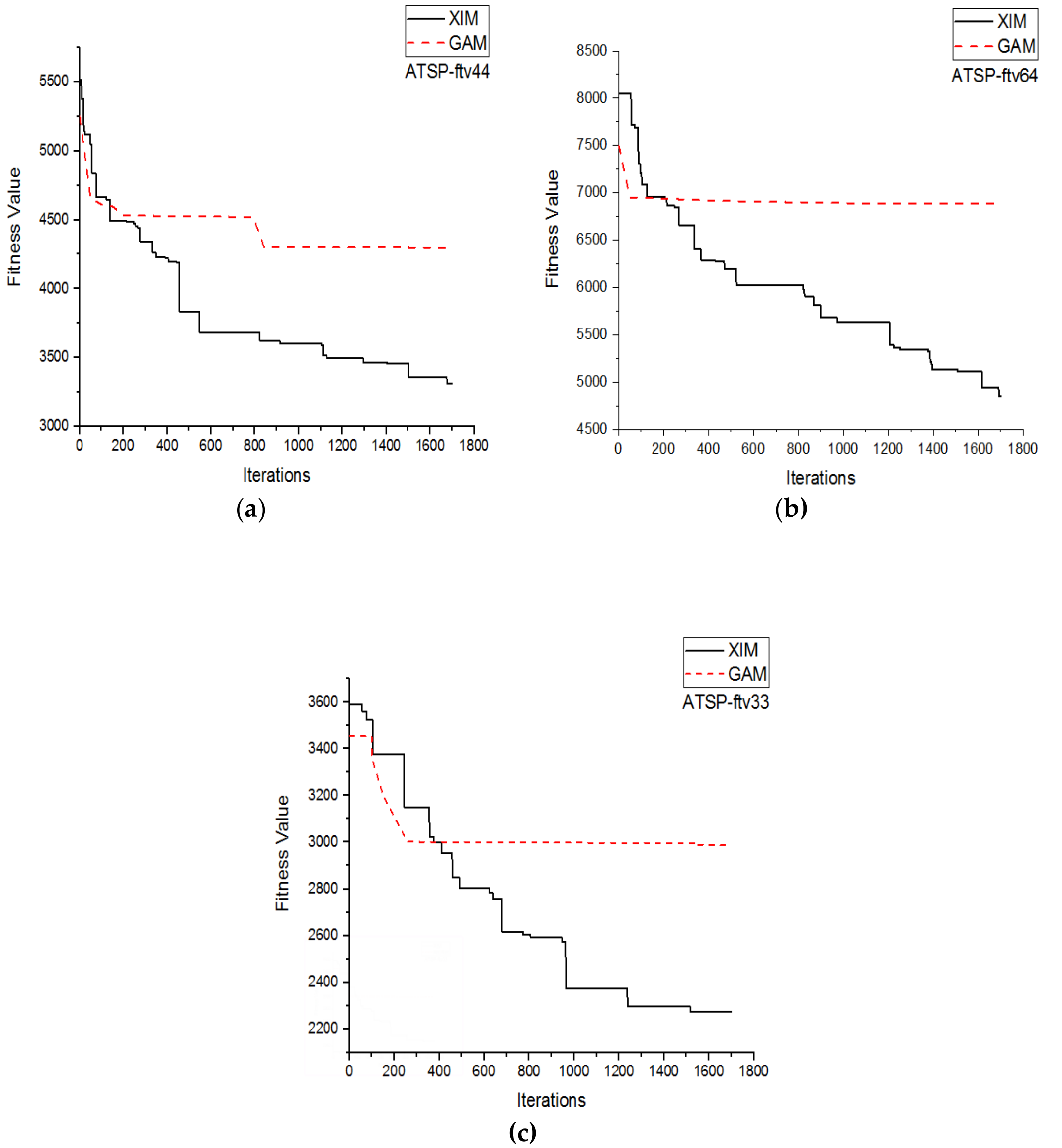


Fig. 4 Global fitness value in XIM and GAM for(**a**) ATSP-FTV 33 ; (**b**) ATSP-FTV 44 ; (**c**) ATSP-FTV 64.

Table 1: Convergence of optimal solution for ATSP-FTV series

	POP 10		POP 20		POP 30	
	Init	**Best**	**Init**	**Best**	**Init**	**Best**
FTV33	3981	2192	3868	2163	3795	2021
FTV35	4209	2875	4274	2832	4698	2559
FTV38	4400	2898	4794	2812	4895	2786
FTV44	5647	3327	5517	3313	5555	3320
FTV47	6281	4105	6237	3835	6176	3305
FTV55	6956	4592	6637	4487	6745	3742
FTV64	8226	5244	8171	4899	8052	4849
FTV70	8980	6138	8932	5769	9116	5604
FTV170	24393	18906	24772	18334	25078	16982

Table 2: Finding the optimum solution for other ATSP series

	POP 10		POP 20		POP 30	
	Init	**Best**	**Init**	**Best**	**Init**	**Best**
BR17	194	41	166	41	152	39
FT53	25386	16860	23822	16516	24246	16032
FT70	69850	58730	68747	57049	67176	56671
KRO124P	175357	125073	179656	118582	176183	101284
P43	17101	5887	17559	5758	17361	5743
RY48P	45071	26766	47553	25885	51734	22268

Table 3: Numerical results for ATSP-RBG series

	POP 10		POP 20		POP 30	
	Init	**Best**	**Init**	**Best**	**Init**	**Best**
RBG323	6073	5557	5978	5555	5787	5465
RBG358	6740	6262	6651	6223	6655	6135
RBG443	7886	7553	8081	7485	8073	7388

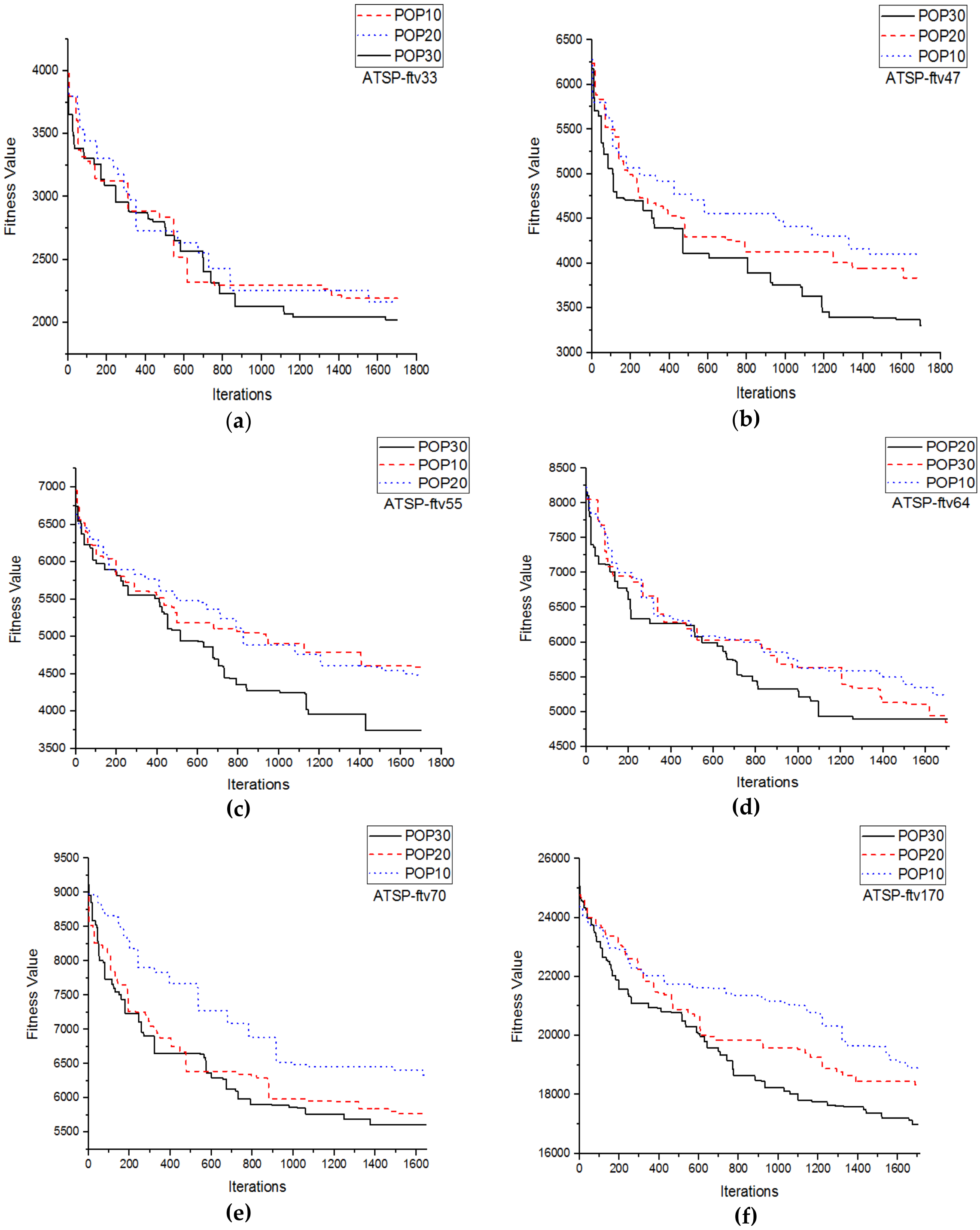


Fig. 5 Curve converging towards the optimum for ATSP instances - FTV: (a) ATSP-FTV33; (b) ATSP-FTV 47; (c) ATSP-FTV55; (d) ATSP-FTV 64; (e) ATSP-FTV 70; (f) ATSP-FTV 170.

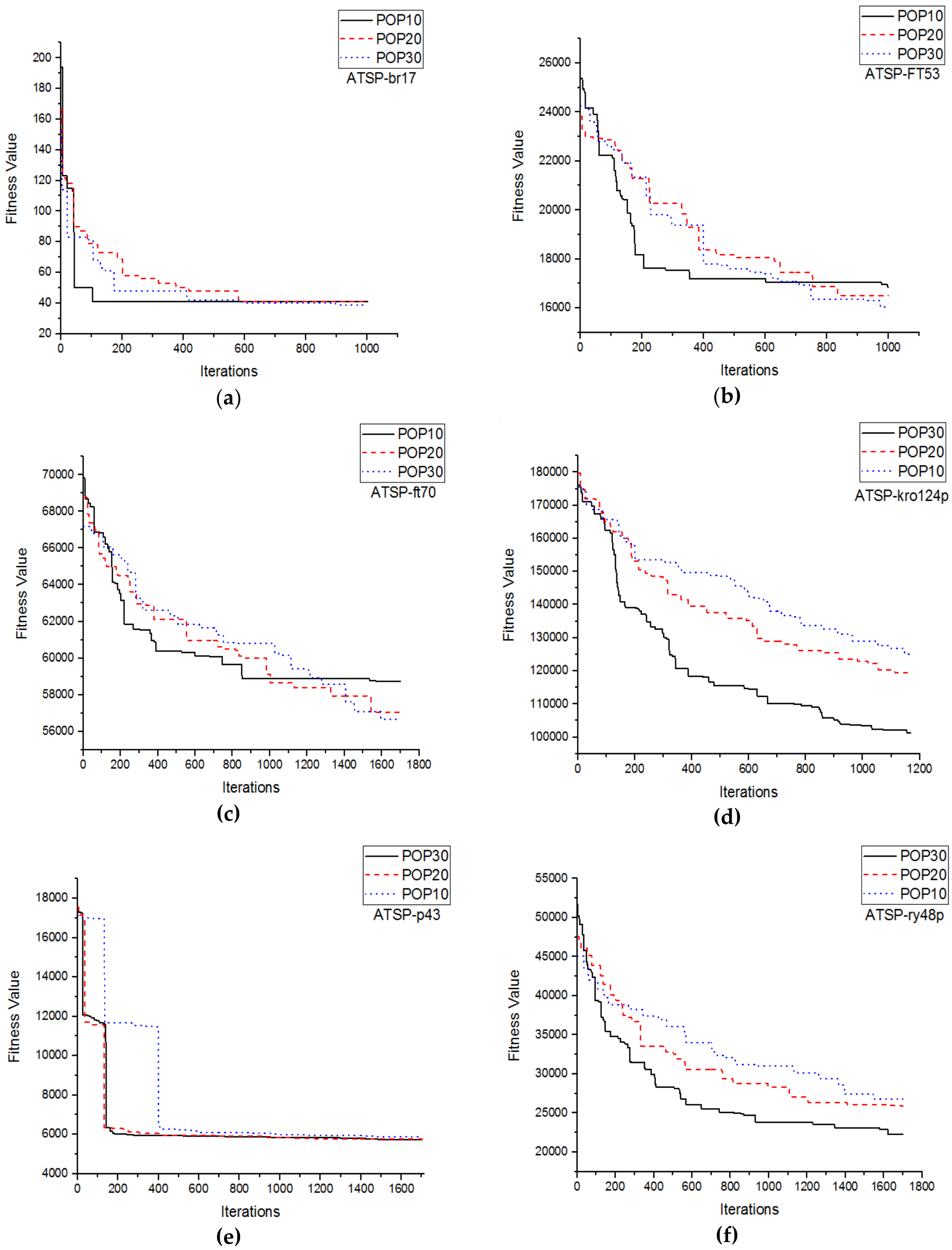
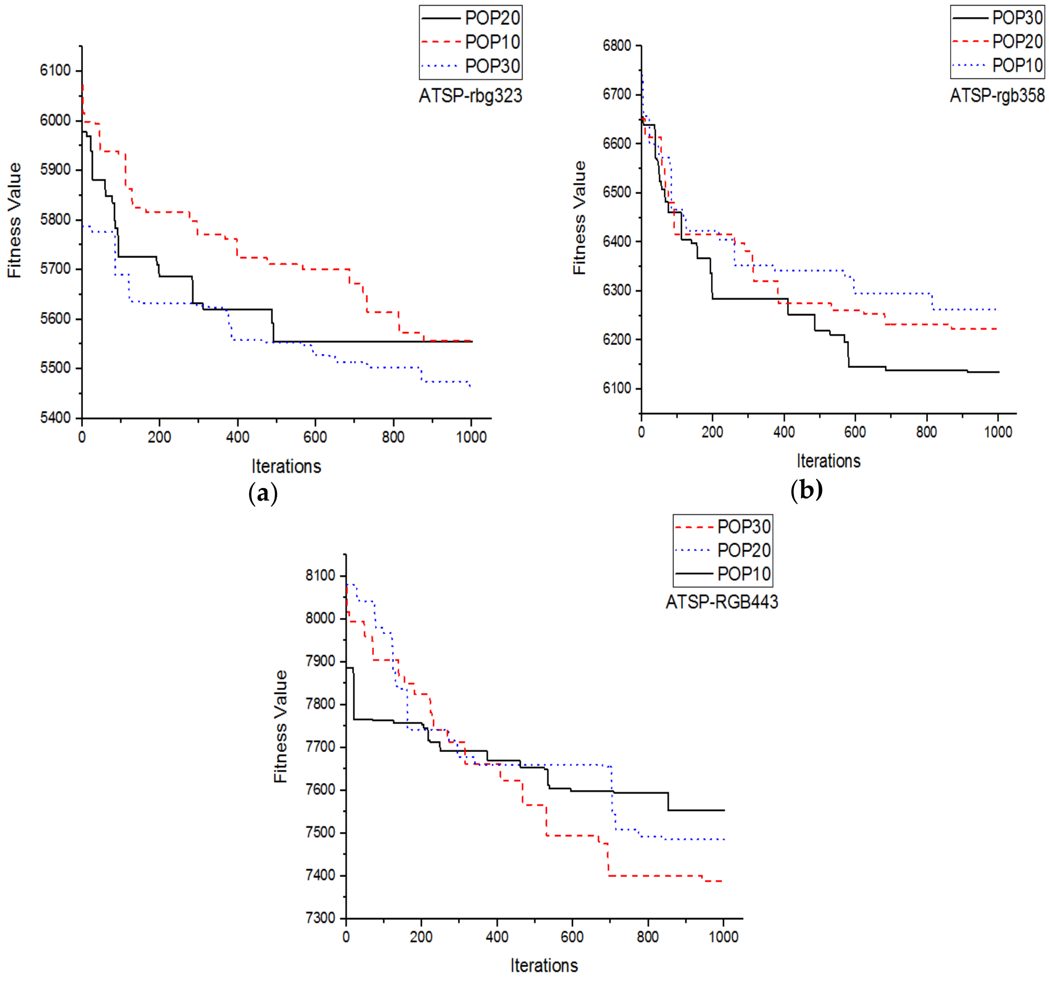


Fig. 6 Converging towards the optimum for others ATSP instances:
(a) ATSP-BR 17; (b) ATSP-FT 53; (c) ATSP-FT 70; (d) ATSP-KRO 124P;
(e) ATSP-P43; (f) ATSP-RY48P.



(a) (b) (c)

Fig. 7 Curve converging towards the optimum for ATSP instances – RGB: (a) ATSP-RGB 323 ; (b) ATSP-RGB358 ; (c) ATSP-RGB443.

5 Conclusion

In this paper, we focused on the possibility to perform a genetic algorithm by adapting the operators involved in this type of method. Thus, an appropriate crossover operator (XIM) for combinatorial problem ATSP is proposed having presented the mathematical formulation of the latter. Numerical results are developed for 18 standard instances taken from Standard asymmetric TSP libraries, showing the effectiveness of the new operator to obtain a better solution.

6 Open Problems

The purpose of transportation problems is to get the overall optimum with less complexity. Thus, it is interesting to be able to hybridize meta-heuristics to benefit from their abilities to improve the optimal solution. Drawing inspiration from the natural phenomenon to introduce new operators allowing diversity and dynamism of the population during the generations also remains a very interesting way to decrease the complexity of the algorithms.

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