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Elzaki transform and the decomposition method for nonlinear fractional partial differential equations

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Abstract

The aim of this paper is to make change to the application of the modified Adomian decomposition method suggested in [11] and we extend to obtaining solutions of nonlinear partial differential equations with time-fractional derivative. The fractional derivative is described in the Caputo sense. Some illustrative examples are given, revealing the effectiveness and convenience of the method.

Keywords*: Adomian decomposition method; Elzaki transform method; Caputo fractional derivative; nonlinear equations.*

1 Introduction

Differential equations are two types: linear differential equations and non-linear differential equations. Non-linear differential equations are the most complex in the solution compared with linear differential equations due to the presence of non-linear part in them. So we find that a lot of researchers are working to develop new methods to solve this kind of equations. These efforts resulted in the consolidation of this research field in many methods, among them we find the Adomian decomposition method. This method was developed from the 1970s to the 1990s by George Adomian ([1]-[5]). Then, a new option emerged recently, includes the composition of Laplace transform, sumudu transform, Natural Transform or Elzaki transform with this method for solving linear and nonlinear differential equations. Among which are the Adomian decomposition method coupled with Laplace transform [6],[7], Adomian decomposition sumudu transform method [8], natural decomposition method [9] and Elzaki transform decomposition algorithm [10],[12].

The motivation of this paper is to extend the application of Elzaki transform decomposition method suggested in [11] to solve nonlinear partial differential equations with time-fractional derivative. The fractional derivative is described in the Caputo sense.

The present paper has been organized as follows: In Section 2 some basic definitions of ELzaki transform method are mentioned. In section 3 we will propose an analysis of the modified method. In section 4 nonlinear time-fractional partial differential equations is studied with the fractional Elzaki transform decomposition method (FETDM). Finally, the conclusion follows.

2 Basic definitions

In this section, we give some basic definitions and properties of fractional calculus, Elzaki transform and Elzaki transform of fractional derivatives which are used further in this paper.

2.1 Fractional calculus.

In There are several definitions of a fractional derivative of order $\alpha \geq 0$ (see [13]-[15]). The most commonly used definitions are the Riemann--Liouville and Caputo. We give some basic definitions and properties of the fractional calculus theory which are used further in this paper.

Definition 2.1 Let Ω=[a,b] ($-\infty<a<b<+\infty$) be a finite interval on the real axis $\mathbb{R}$. The Riemann-Liouville fractional integrals $I^{\alpha} f$ of order $\alpha \in \mathbb{R}$ ($\alpha>0$) is defined by

$$(1) \qquad (I_{0+}^{\alpha} f)(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, (t>0, \alpha>0).$$

Here $\Gamma(\cdot)$ is the well-known Gamma function.

Theorem 2.2 Let ($\alpha \geq 0$) and let $n=[\alpha]+1$. If $f(t) \in AC^n[a,b]$ then the Caputo fractional derivatives $({}^cD^{\alpha} f)(t)$ exist almost everywhere on [a,b].

If $\alpha \notin \mathbb{N}$, $({}^cD^{\alpha} f)(t)$ are represented by :

$$(2) \qquad ({}^cD_{0+}^{\alpha} f)(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha+1} f(\tau) d\tau,$$

where D=d/dx and $n=[\alpha]+1$.

Proof (see [14]).

Remark 2.1 In this paper, we consider the time-fractional derivative in the Caputo's sense. When $\alpha \in \mathbb{R}^+$, the time-fractional derivative is defined as :

$$
{}^{c}D_t^{\alpha}u(x,t)=\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}
$$

(3)
$$
=\begin{cases}\dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_0^t (t-\tau)^{m-\alpha-1}\frac{\partial^m u(x,\tau)}{\partial\tau^m}d\tau, 0<m-1<\alpha<m,\\ \dfrac{\partial^m u(x;t)}{\partial t^m}, \alpha=m,\end{cases}
$$

where m ϵ $\mathbb{N}^*$.

2.2 Definitions of Elzaki transform

A new integral transform called Elzaki transform ([16]-[18]) defined for functions of exponential order, is proclaimed. They consider functions in the set *A* defined by :

(4)
$$
A=\left\{ f(t): M, k_1, k_2>0, |f(t)|<Me^{\frac{t}{k_j}}, ift \in (-1)^j\times[0,\infty) \right\}.
$$

Definition 2.3 If f(t) is function defined for all t≥0, its Elzaki transform is the integral of f(t) times –t/s from t=0 to ∞. It is a function of s and is defined by E[f]

(5)
$$
E[f(t)]=T(s)=s\int_0^{\infty} f(t)e^{-\frac{t}{s}}dt.
$$

Theorem 2.4 Elzaki transform amplifies the coefficients of the power series function :

(6)
$$
f(t)=\sum_{n=0}^{\infty}a_n t^n,
$$

on the new integral transform "Elzaki transform", given by :

(7)
$$
E[f(t)]=T(v)=\sum_0^{\infty} n!a_n v^{n+2}.
$$

Theorem 2.5 Let f(t) be in *A* and Let $T_n(v)$ denote Elzaki transform of *nth* derivative $f^{(n)}$(t) of f(t), then for n≥1,

$$(8)\qquad T_n(v)=\frac{T(v)}{v^n}=\sum_{k=0}^{n-1}v^{2-n+k}f^{(k)}(0).$$

To obtain Elzaki transform of partial derivative we use integration by parts, and then we have :

$$(9)\qquad E\left(\frac{\partial f(x,t)}{\partial t}\right)=\frac{1}{v}T(x,v)-vf(x,0),$$

$$(10)\qquad E\left(\frac{\partial^2 f(x,t)}{\partial t^2}\right)=\frac{1}{v^2}T(x,v)-f(x,0)-v\frac{\partial f(x,0)}{\partial t}.$$

Properties of Elzaki transform can be found in Refs.([16],(17]), we mention only the following :

$1.E(1)=v^2;$ $\qquad$ $3.E(t^n)=n!v^{n+2};$

$2.E(t)=v^3;$ $\qquad$ $4.E(v^{n+2})=\dfrac{t^n}{n!}.$

2.3 Elzaki transform of fractional derivatives

To give the formula of Elzaki transform of Caputo fractional derivative, we use the Laplace transform formula for the Caputo fractional derivative [13].

$$(11)\qquad L\left\{{}^cD_{0+}^{\alpha}f(t);s\right\}=s^{\alpha}F(s)-\sum_{k=0}^{m-1}s^{\alpha-k-1}f^{(k)}(0),$$

where (m-1<α≤m) m ϵ $\mathbb{N}^*$.

Theorem 2.6 [19] Let *A* defined as above. With Laplace transform F(s), then the Elzaki transform *T(v)* of *f*(t) is given by :

$$(12)\qquad T(v)=vF\left(\frac{1}{v}\right).$$

Theorem 2.7 Suppose *T(v)* is the Elzaki transform of the function *f*(t) then :

$$(13)\qquad E\left\{{}^cD_{0+}^{\alpha}f(t);v\right\}=\frac{T(v)}{v^{\alpha}}-\sum_{k=0}^{n-1}s^{k-\alpha+2}f^{(k)}(0).$$

Proof (see [20]).

3 Fractional Elzaki decomposition method (FEDM)

In this section, we make a change to the method proposed in [11] and we extend to solving nonlinear partial differential equations with time-fractional derivative. To illustrate the basic idea of this method, we consider a general nonlinear nonhomogeneous fractional partial differential equation :

$$(14)\qquad {}^{C}D_t^{\alpha} w(x,t) + Rw(x,t) + Nw(x,t) = g(x,t),$$

where m-1<α≤m, m=1,2,…, and the initial conditions :

$$(15)\qquad \left[\frac{\partial^{m-1} w(x,t)}{\partial^{m-1} t}\right]_{t=0} = f_{m-1}(x), m = 1,2,...,$$

where ${}^{c}D_t^{\alpha} w(x,t)$ is the Caputo fractional derivative of the function *w(x,t), R* is the linear differential operator, *N* represents the general nonlinear differential operator, and *g(x,t)* is the source term.

Applying Elzaki transform (denoted in this paper by *E*) on both sides of Eq. (14), we get :

$$(16)\qquad E[{}^{c}D_t^{\alpha} w(x,t)] + E[Rw(x,t)] + E[Nw(x,t)] = E[g(x,t)].$$

Using the property of Elzaki transform, we obtain :

$$(17)\qquad \begin{aligned} E[w] = \sum_{k=0}^{m-1} v^{k+2}\left[\frac{\partial^k w}{\partial t^k}\right]_{t=0} + v^{\alpha} E[g(x,t)] \\ - v^{\alpha} E\left[Rw + Nw\right], \end{aligned}$$

where m-1<α≤m, m=1,2,….

Operating with the inverse Elzaki transform on both sides of Eq. (17), we get :

$$(18)\qquad w(x,t) = G(x,t) - E^{-1}(v^{\alpha} E[Rw(x,t) + Nw(x,t)]),$$

where *G(x,t),* represents the term arising from the source term and the prescribed initial conditions. The second step in Elzaki decomposition method, is that we represent solution as an infinite series given below:

$$(19)\qquad w(x,t) = \sum_{n=0}^{\infty} w_n(x,t),$$

and the nonlinear term can be decomposed as:

$$(20)\qquad Nw(x,t) = \sum_{n=0}^{\infty} A_n(w),$$

where A_n are He's polynomials [21] of w_0, w_1, w_2, ...,w_n, which can be calculated by the following formula :

$$A_n(w_0,...,w_n) = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[N(\sum_{i=0}^{\infty}\lambda^i w_i)\right]_{p=0} \quad ,n = 0,1,2,... \tag{21}$$

Substituting (19) and (20) in (18), we have:

$$\sum_{n=0}^{\infty} w_n = G(x,t) - E^{-1}(v^{\alpha}E[R\sum_{n=0}^{\infty} w_n + N\sum_{n=0}^{\infty} A_n(w)]), \tag{22}$$

On comparing both sides of Eq. (20), we get:

$$\begin{aligned} &w_0(x,t) = G(x,t), \\ &w_1(x,t) = -E\left(v^{\alpha}E\left[Rw_0(x,t) + A_0(w)\right]\right), \\ &w_2(x,t) = -E\left(v^{\alpha}E\left[Rw_1(x,t) + A_1(w)\right]\right), \end{aligned} \tag{23}$$

In general, the recursive relation is given as :

$$w_{n+1}(x,t) = -E^{-1}(v^{\alpha}E[Rw_n + A_n(w)]), n \geq 0. \tag{24}$$

Finally, we approximate the analytical solution by truncated series:

$$w(x,t) = \lim_{N\to\infty}\sum_{n=0}^{N} w_n(x,t). \tag{25}$$

The above series solutions generally converge very rapidly [22].

4 Application of the FETDM

In this section, we apply the fractional Elzaki transform decomposition method (FETDM) for the Caputo fractional derivative, to solve nonlinear partial differential equation with time-fractional derivative.

Example 4.1 Consider the following time-fractional partial differential equation :

$${}^cD_t^{\alpha}w + ww_x - w_{xx} = 0, 0 < \alpha \leq 1, \tag{26}$$

subject to the initial condition

$$w(x,0) = x. \tag{27}$$

Applying Elzaki transform on both sides of Eq. (26). Thus, we get:

$$E[{}^cD_t^\alpha w]+E[ww_x]-E[u_{xx}]=0. \tag{28}$$

Using the differentiation property of Elzaki transform, we have :

$$E[w(x,t)]=x-E^{-1}(v^\alpha E[ww_x-w_{xx}]. \tag{29}$$

By applying the aforesaid decomposition method, we find :

$$\sum_{n=0}^{\infty} w_n(x,t)=x-E^{-1}(v^\alpha E[\sum_{n=0}^{\infty} A_n(w)-\sum_{n=0}^{\infty}(w_n)_{xx}]), \tag{30}$$

The first few components of *$A_n(w)$* polynomials [21] for example, is given by :

$$\begin{aligned} &A_0(w)=w_0w_{0x},\\ &A_1(w)=w_0w_{1x}+w_1w_{0x},\\ &A_2(w)=w_0w_{2x}+w_2w_{0x}+w_1w_{1x},\\ &\vdots \end{aligned} \tag{31}$$

On comparing both sides of Eq. (30), we get:

$$\begin{aligned} &w_0(x,t)=x,\\ &w_1(x,t)=-E^{-1}\left(v^\alpha E\left[A_0(w)-(w_0)_{xx}\right]\right),\\ &w_2(x,t)=-E^{-1}\left(v^\alpha E\left[A_1(w)-(w_1)_{xx}\right]\right),\\ &w_3(x,t)=-E^{-1}\left(v^\alpha E\left[A_2(w)-(w_2)_{xx}\right]\right),\\ &\vdots \end{aligned} \tag{32}$$

Using He's polynomials (31) and the iteration formulas (32), we obtain:

$$\begin{aligned} &w_0(x,t)=x,\\ &w_1(x,t)=-x\frac{1}{\Gamma(\alpha+1)}t^\alpha,\\ &w_2(x,t)=x\frac{2}{\Gamma(2\alpha+1)}t^{2\alpha},\\ &w_3(x,t)=-x\frac{4[\Gamma^2(\alpha+1)+\Gamma(2\alpha+1)]}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\\ &\vdots \end{aligned} \tag{33}$$

The approximate solution of Eq. (26), is given by:

$$w(x,t)=x-x\frac{1}{\Gamma(\alpha+1)}t^{\alpha}+x\frac{2}{\Gamma(2\alpha+1)}t^{2\alpha}$$

$$-x\frac{4(\Gamma(\alpha+1))^2+\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2\Gamma(3\alpha+1)}t^{3\alpha}+\cdots \tag{34}$$

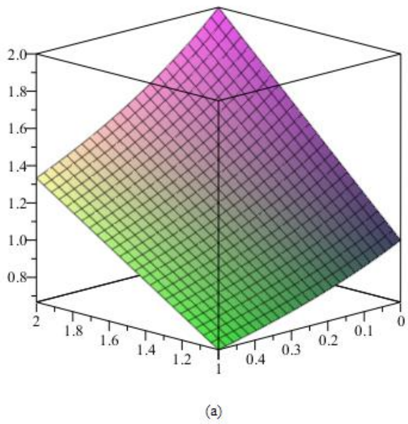
The approximate solution of Eq. (26) in the special case α=1, is given by

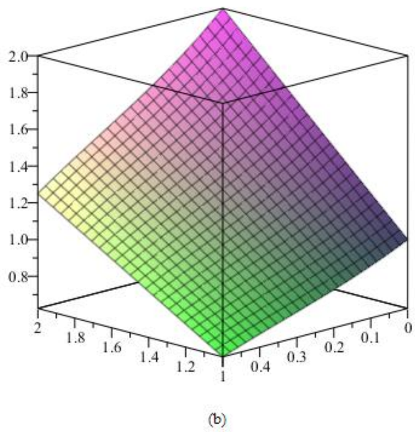
$$w(x,t)=x(1-t+t^2-t^3+t^4\cdots). \tag{35}$$

That gives :

$$w(x,t)=\frac{x}{1+t},|t|<1, \tag{36}$$

which is an exact solution of KdV equation as presented in [23].





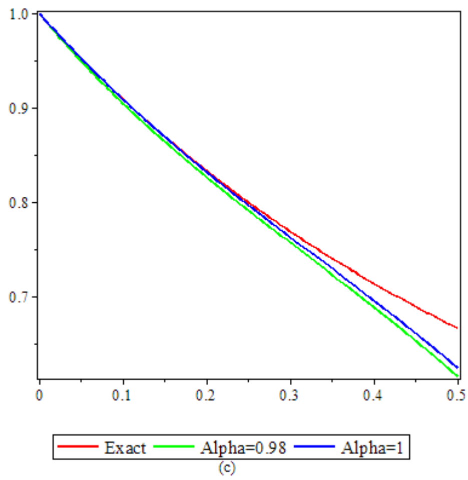


Fig. 1 *:* (a) Exact solution, (b) the approximate solution in the case α=1, (c) The exact solution and approximate solutions of Eq. (23) for different values of α when x=1.

Example 4.2 Consider the nonlinear time-fractional partial differential equation of order α (1<α≤2)

$$ {}^{c}D_t^{\alpha}w+\frac{2}{t}ww_x=0,t>0,x\neq 0, \tag{37}$$

with the initial conditions

$$w(x,0)=0, w_t(x,0)=\frac{1}{x}. \tag{38}$$

If α=2 and with the initial conditions (38), the exact solution of the following equation :

$$w_{tt}+\frac{2}{t}ww_x=0, t>0, x\neq 0, \tag{39}$$

is given by :

$$w(x,t)=\tan\left(\frac{t}{x}\right). \tag{40}$$

By using (22), we can obtain the iteration formula :

$$\sum_{n=0}^{\infty} w_n(x,t)=\frac{t}{x}-E^{-1}\left(v^{\alpha}E\left[\frac{2}{t}\sum_{n=0}^{\infty}A_n(w)\right]\right). \tag{41}$$

The first few components of $A_n(w)$ polynomials [21] for example, is given by:

$$\begin{aligned} &A_0(w)=w_0w_{0x}, \\ &A_1(w)=w_0w_{1x}+w_1w_{0x}, \\ &A_2(w)=w_0w_{2x}+w_2w_{0x}+w_1w_{1x}, \\ &\vdots \end{aligned} \tag{42}$$

In a similar way as above, we have:

$$\begin{aligned} &w_0(x,t)=\frac{t}{x}, \\ &w_1(x,t)=\frac{2}{\Gamma(\alpha+2)}\frac{t^{\alpha+1}}{x^3}, \\ &w_2(x,t)=\frac{16}{\Gamma(2\alpha+2)}\frac{t^{2\alpha+1}}{x^5}, \\ &w_{23}(x,t)=\frac{24}{\Gamma(3\alpha+2)}\left[8+\frac{\Gamma(2\alpha+2)}{\Gamma^2(\alpha+2)}\right]\frac{t^{3\alpha+1}}{x^7}, \\ &\vdots \end{aligned} \tag{43}$$

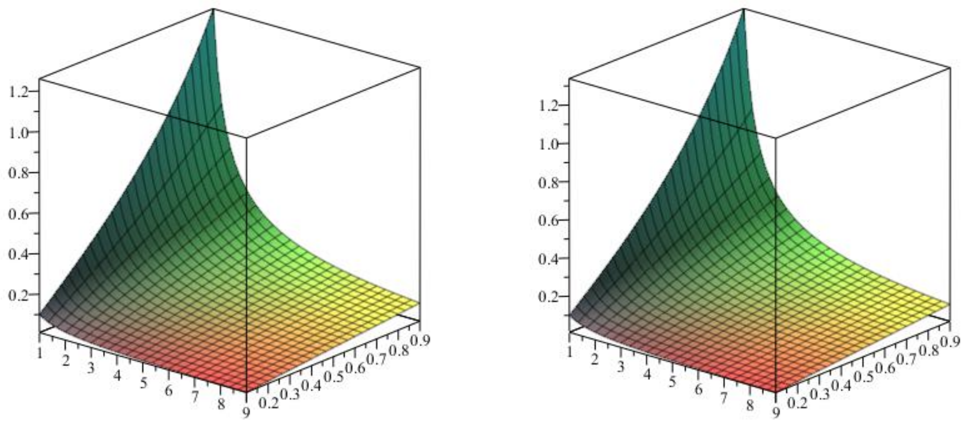
The approximate solution in a series form, is given by :

$$w(x,t) = \frac{t}{x} + \frac{2}{\Gamma(\alpha+2)}\frac{t^{\alpha+1}}{x^3} + \frac{16}{\Gamma(2\alpha+2)}\frac{t^{2\alpha+1}}{x^5}$$
$$\frac{24}{\Gamma(3\alpha+2)}\left[8 + \frac{\Gamma(2\alpha+2)}{(\Gamma(\alpha+2))^2}\right]\frac{t^{3\alpha+1}}{x^7} + \cdots \tag{44}$$

As α=2, we obtain :

$$w(x,t) = \frac{t}{x} + \frac{1}{3}\left(\frac{t}{x}\right)^3 + \frac{2}{15}\left(\frac{t}{x}\right)^5 + \frac{17}{315}\left(\frac{t}{x}\right)^7 = \tan\left(\frac{t}{x}\right) \tag{45}$$

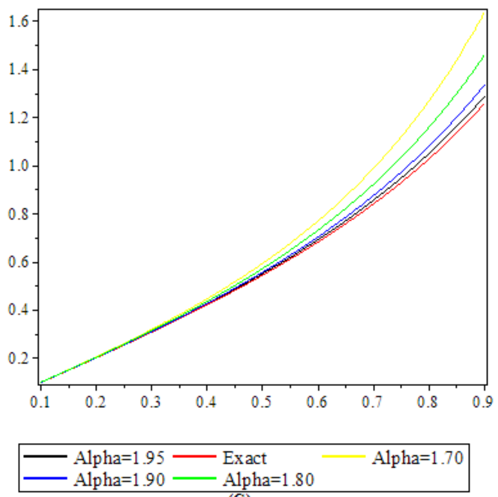
which is an exact solution of the nonlinear partial differential equation (34).



(A) (B)

Remark For graphs of the approximate solution, we took only four terms in the two previous examples.

Fig. 2 : **(A)** The exact solution, **(B)** The approximate solution when α=2.90, **(C)** The approximate solutions of Eq. (32) for different values of α when x=1.



(C)

Example 4.3 Consider the nonlinear time-fractional partial differential equation of order α (2<α≤3)

$$ {}^{c}D_t^{\alpha} w - 3ww_{xx} = 0, t > 0, x \neq 0, \tag{46}$$

with the initial conditions

$$ w(x,0) = \frac{1}{x}, w_t(x,0) = \frac{1}{x^2}, w_{tt}(x,0) = \frac{2}{x^3}. \tag{47}$$

If α=3 and with the initial conditions (47), the exact solution of the following equation :

$$ w_{ttt} - 3ww_x = 0, t > 0, x \neq 0, \tag{48}$$

is given by :

$$ w(x,t) = \frac{1}{x-t}, \left|\frac{t}{x}\right| < 1, x \neq 0. \tag{49}$$

By using we can obtain the iteration formula :

$$ \sum_{n=0}^{\infty} w_n(x,t) = \frac{1}{x} + \frac{t}{x^2} + \frac{t^2}{x^3} + 3E^{-1}\left(v^{\alpha} E\left[\sum_{n=0}^{\infty} A_n(w) \right] \right), \tag{50}$$

The first few components of $A_n(w)$ polynomials [21] for example, is given by:

$$ \begin{aligned} & A_0(w) = w_0 w_{0x}, \\ & A_1(w) = w_0 w_{1x} + w_1 w_{0x}, \\ & A_2(w) = w_0 w_{2x} + w_2 w_{0x} + w_1 w_{1x}, \\ & \vdots \end{aligned} \tag{51}$$

In a similar way as above, we have :

$$ w_0(x,t) = \frac{1}{x}\left[1 + \frac{t}{x} + \frac{t^2}{x^2} \right], $$

$$ w_1(x,t) = \frac{1}{x}\left[a_1 \frac{t^{\alpha}}{x^3} + a_2 \frac{t^{\alpha+1}}{x^4} + a_3 \frac{t^{\alpha+2}}{x^5} + a_4 \frac{t^{\alpha+3}}{x^6} + a_5 \frac{t^{\alpha+4}}{x^7} \right], $$

$$w_2(x,t) = \frac{1}{x}\left[a_6 \frac{t^{2\alpha}}{x^6} + a_7 \frac{t^{2\alpha+1}}{x^7} + a_8 \frac{t^{2\alpha+2}}{x^8} + a_9 \frac{t^{2\alpha+3}}{x^9} + a_{10} \frac{t^{2\alpha+4}}{x^{10}} \right],$$

$$\vdots$$

where :

$$a_1 = \frac{3!}{\Gamma(\alpha+1)}, a_2 = \frac{4!}{\Gamma(\alpha+2)}, a_3 = \frac{5!}{\Gamma(\alpha+3)}, a_4 = \frac{2^2 \times 3^3}{\Gamma(\alpha+4)},$$

$$a_5 = \frac{2^5 \times 3^3}{\Gamma(\alpha+5)}, a_6 = \frac{66 \times 3!}{\Gamma(2\alpha+1)}, a_7 = \left[\frac{78 \times 3!}{\Gamma(\alpha+1)} + \frac{96 \times 4!}{\Gamma(\alpha+2)} \right] \frac{\Gamma(\alpha+2)}{\Gamma(2\alpha+2)},$$

$$a_8 = \left[\frac{24 \times 4!}{\Gamma(\alpha+1)} + \frac{108 \times 4!}{\Gamma(\alpha+2)} + \frac{132 \times 5!}{\Gamma(\alpha+3)} \right] \frac{\Gamma(\alpha+3)}{\Gamma(2\alpha+3)},$$

$$a_9 = \left[\frac{126 \times 4!}{\Gamma(\alpha+2)} + \frac{24 \times 6!}{\Gamma(\alpha+3)} \right] \frac{\Gamma(\alpha+4)}{\Gamma(2\alpha+4)}, a_{10} = \frac{18 \times 6!}{\Gamma(\alpha+3)} \times \frac{\Gamma(\alpha+5)}{\Gamma(2\alpha+5)}.$$

The first terms of the approximate solution of Eq. (46), is given by :

$$w(x,t) = \frac{1}{x}\left[1 + \frac{t}{x} + \frac{t^2}{x^2} + a_1 \frac{t^{\alpha}}{x^3} + a_2 \frac{t^{\alpha+1}}{x^4} + a_3 \frac{t^{\alpha+2}}{x^5} + a_4 \frac{t^{\alpha+3}}{x^6} + a_5 \frac{t^{\alpha+4}}{x^7} \right.$$
$$\left. + a_6 \frac{t^{2\alpha}}{x^6} + a_7 \frac{t^{2\alpha+1}}{x^7} + a_8 \frac{t^{2\alpha+2}}{x^8} + a_9 \frac{t^{2\alpha+3}}{x^9} + a_{10} \frac{t^{2\alpha+4}}{x^{10}} + \cdots \right].$$

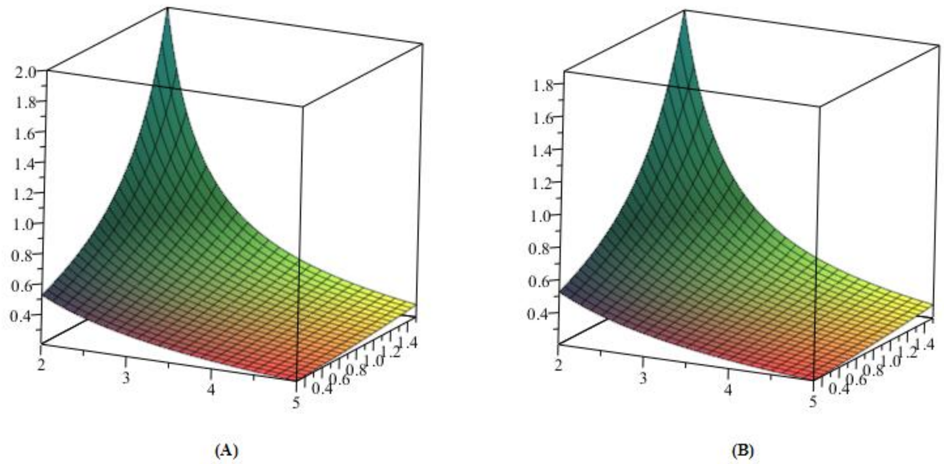
As α=3, we obtain :

$$w(x,t) = \frac{1}{x}\left[1 + \frac{t}{x} + \left(\frac{t}{x}\right)^2 + \left(\frac{t}{x}\right)^3 + \left(\frac{t}{x}\right)^4 + \left(\frac{t}{x}\right)^5 \right.$$
$$\left. \left(\frac{t}{x}\right)^6 + \left(\frac{t}{x}\right)^7 + \left(\frac{t}{x}\right)^8 + \frac{15}{28}\left(\frac{t}{x}\right)^9 + \frac{3}{20}\left(\frac{t}{x}\right)^{10} + \cdots \right]. \quad (52)$$

That gives :

$$w(x,t) = \frac{1}{x-t}, \left|\frac{t}{x}\right| < 1, x \neq 0, \quad (49)$$

which is an exact solution of the nonlinear partial differential equation (48) as presented in [11].



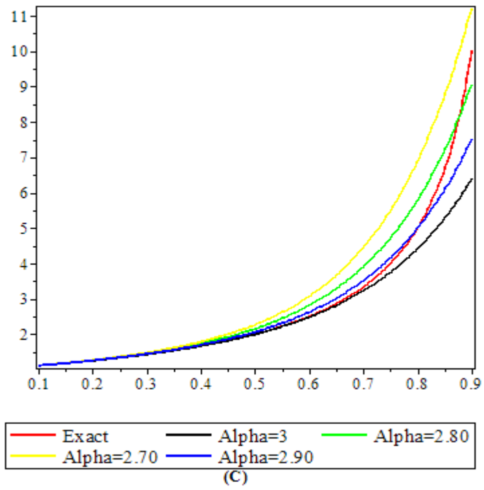


Fig. 3 : **(A)** The exact solution, **(B)** The approximate solution when $\alpha = 3$, **(C)** The approximate solutions of Eq. (40) for different values of α when $x = 1$, and we took only three terms.

5 Conclusion

In this paper, the fractional Elzaki transform decomposition method (FETDM) has been applied for finding the exact or the approximate solutions to the nonlinear fractional partial differential equations. The FETDM can easily be applied to many problems and is capable of reducing the size of computational work. The results show that the fractional Elzaki transform decomposition method (FETDM) is an appropriate method for solving nonlinear partial differential equations of fractional orders.

6 Open Problem

In this work, the fractional Elzaki transform decomposition method (FETDM) to be effective for solving nonlinear partial differential equation with time-fractional derivative. One can apply the fractional natural transform decomposition method (FNTDM),...,to the same problem (nonlinear fractional partial differential equations). Is it possible to solve nonlinear partial integro-differential equations of fractional order by this method (FETDM)?

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