

Heat and mass transfer effects on an unsteady hydromagnetic free convective flow over an infinite vertical plate embedded in a porous medium with heat absorption

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Abstract

An unsteady, two dimensional, hydro magnetic, laminar mixed convective boundary layer flow of an incompressible and electrically conducting fluid along an infinite vertical plate embedded in the porous medium with heat and mass transfer is analyzed, by taking into account the effects of viscous dissipation and heat absorption. The dimensionless governing equations for this investigation are solved analytically using Galerkin finite element method. Numerical evaluation of the numerical results is performed and the results for velocity, temperature and concentration profiles within the boundary layer are discussed and shown through graphically.

Keywords: *MHD, unsteady, free convection flow, FEM, Vertical plate*

1 Introduction

Combined buoyancy – generated heat and mass transfer due to temperature and concentration variations, in fluid – saturated porous media, have several important applications in variety of engineering processes including heat exchanger devices, petroleum reservoirs, chemical catalytic reactors, solar energy porous wafer collector systems, ceramic materials, migration of moisture through air contained in fibrous insulations and grain storage installations and the dispersion of

chemical contaminants through water – saturated soil, super convecting geothermic etc. Ahmed and Kalita [1] have investigated the effect of the thermal diffusion as well as magnetic field on free convection and mass transfer flow through porous medium, taking into account the effect of a of heat source. Anjali Devi and Kayalvizhi [2] presented analytical solution of MHD flow with radiation over a stretching sheet embedded in a porous medium. Ayani and Fsfahani [3] studied the effect of thermal radiation on an unsteady magnetohydrodynamic free convection flow past an infinite vertical porous plate in presence of heat source. The effects of radiation on a steady combined free – forced convective and mass transfer flow of a viscous incompressible electrically conducting and radiating fluid over an isothermal semi – infinite vertical porous flat plate embedded in a porous medium studied by Bala Anki Reddy and Bhaskar Reddy [4]. Influence of thermal radiation on transient magnetohydrodynamic Couette flow through a porous medium by using finite difference method discussed by Baoku *et al.* [5]. Beg *et al.* [6] used the network thermodynamic simulation approach to study the hydromagnetic convection flow from an isothermal sphere to a non – Darcian porous medium with heat generation or absorption effects. Chamka and Ahmed [7] found the similarity solution for an unsteady magnetohydrodynamic flow near a stagnation point of a three – dimensional porous body with heat and mass transfer, heat generation/absorption and chemical reaction. A study of Hall effects over the heat and mass transfer flow of visco – elastico fluid is made by Chaudhary *et al.* [8]. Chaudhary *et al.* [9] have considered the effect of radiation on MHD heat transfer past vertical plate. Deka and Bhattacharya [10] obtained an exact solution of unsteady free convective Couette flow of a viscous incompressible heat generating/ absorbing fluid confined between two vertical plates in a porous medium. Ganeswara Reddy and Bhaskar Reddy [11] presented Soret and Dufour effects on steady MHD free convection flow past a semi – infinite moving vertical plate in a porous medium with viscous dissipation.

Recently, Hemant Poonia and Chaudhary [12] studied an unsteady, two – dimensional, hydromagnetic, laminar mixed convective boundary layer flow of an

incompressible and electrically conducting fluid along an infinite vertical plate embedded in the porous medium with heat and mass transfer is analyzed, by taking into account the effects of viscous dissipation. Analytical solutions for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting and heat generation/absorbing fluid on a continuously vertical permeable surface in the presence of a radiation, a first – order homogeneous chemical reaction and the mass flux are reported by Kesavaiah *et al.* [13]. The numerical study of thermal radiation effects on the transient hydromagnetic natural convection flow past a vertical plate embedded in a porous medium with mass diffusion and fluctuating temperature about time at the plate, by taking into account the heat due to viscous dissipation studied by Kishore *et al.* [14]. The natural convection in unsteady Couette flow of a viscous incompressible fluid confined between two vertical parallel plates in the presence of thermal radiation has been studied by Narahari [15]. Prasad *et al.* [16] studied the radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate embedded in porous medium: a numerical study. Rajput and Pradeep [17] have studied the effect of a uniform transverse magnetic field on the unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel plates with constant temperature and Variable mass diffusion. Samad and Mobeujjaman [18] reported the MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation. Sankar Reddy *et al.* [19] studied the effect of thermal radiation on unsteady two – dimensional laminar flow of a viscous incompressible electrically conducting micropolar fluid past a semi – infinite vertical porous moving plate taking into account the effect of magnetic field in the presence of heat absorption. The Rosseland approximation is used to describe radiative heat transfer in the limit of optically thick fluids. A uniform magnetic field acts perpendicular to the porous surface, which absorbs the fluid with a suction velocity varying with time. Seth *et al.* [20] have studied unsteady MHD Couette flow of a viscous incompressible electrically conducting

fluid, in the presence of a transverse magnetic field, between two parallel porous plates. Suneetha *et al.* [21] investigated radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with viscous dissipation. M.Bottarelli [22], Analysed heat transfer induced by an horizontal ground heat exchanger in heterogeneous soil. S.Abhshek *et al.* [23] discussed Magneto hydrodynamic thermo solutal buoyancy darcy convection in a square enclosure.

The objective of the present paper is to analyze the heat and mass transfer effects on an unsteady two dimensional laminar mixed convective boundary layer flow of viscous, incompressible, electrically conducting fluid, along a vertical plate with suction, embedded in porous medium, in the presence of transverse magnetic field, by taking into account the effects of the viscous dissipation and heat absorption. The equation of continuity, motion, energy and mass transfer, which govern the flow field are solved by using a finite element method which is more economical from computational view point and the results obtained are good agreement with the results of Hemant Poonia and Chaudhary [12] in some special cases.

2 Mathematical formulation

An unsteady two – dimensional hydromagnetic laminar mixed convective boundary layer flow of a viscous incompressible electrically conducting fluid past an infinite vertical flat plate in a uniform porous medium, in the presence of heat source, thermal and concentration buoyancy effects has been considered.

The equations governing the flow are:

Equation of Continuity:

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

Momentum Equation:

$$\nu \frac{\partial^2 u'}{\partial y'^2} - v' \frac{\partial u'}{\partial y'} = \frac{\partial u'}{\partial t'} - g\beta(T' - T'_\infty) - g\beta^*(C' - C'_\infty) + \left(\frac{\sigma B_o^2}{\rho} + \frac{\nu}{K'} \right) u' \quad (2)$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u'}{\partial y'} \right)^2 + Q'(T' - T'_\infty) \quad (3)$$

Species Diffusion Equation:

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

The boundary conditions for the velocity, temperature and concentration fields are:

$$t' \leq 0: u' = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } y' \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \quad (5)$$

$$t' > 0: \left\{ \begin{array}{l} u' = 0, T' = T'_\infty + T_o(t)(T'_w - T'_\infty), \\ C' = C'_\infty + C_o(t)(C'_w - C'_\infty) \text{ at } y' = 0 \\ u' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \end{array} \right.$$

Introducing the following non – dimensional parameters in equations (2), (3) and (4) quantities:

$$\left. \begin{array}{l} y = \frac{y'v_o}{\nu}, t = \frac{t'v_o^2}{4\nu}, u = \frac{u'}{v_o}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\ Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{v_o^3}, Gc = \frac{g\beta^* \nu(C'_w - C'_\infty)}{v_o^3}, Pr = \frac{\mu C_p}{\kappa}, \\ M = \left(\frac{\sigma B_o^2}{\rho} \right) \frac{\nu}{v_o^2}, Ec = \frac{v_o^2}{C_p(T'_w - T'_\infty)}, Sc = \frac{\nu}{D}, \\ Q = \frac{\nu Q'}{\rho C_p v_o^2}, K = \frac{K' v_o^2}{\nu^2}, T_o(t) = 1 + \varepsilon e^{i\omega t}, \omega = \frac{4\nu\omega'}{v_o^2}, C_o(t) = 1 + \varepsilon e^{i\omega t} \end{array} \right\} \quad (6)$$

From equation of continuity (1), it is clear that the suction velocity normal to the plate is either a constant or a function of the time. Hence, it is assumed in the form $v' = -v_o(1 + \varepsilon\alpha e^{i\omega t})$ (7)

Where ε and $\varepsilon\alpha$ are small less than unity and v_o is a non – zero positive constant suction velocity, the negative sign indicates that the suction is towards the plate. In terms of (6), equations (2), (3) and (4) become

$$\frac{\partial^2 u}{\partial y^2} + (1 + \varepsilon\alpha e^{i\omega t}) \frac{\partial u}{\partial y} = \frac{1}{4} \frac{\partial u}{\partial t} - (Gr)\theta - (Gc)\phi + (M + \frac{1}{K})u \quad (8)$$

$$\frac{\partial^2 \theta}{\partial y^2} + (Pr)(1 + \varepsilon\alpha e^{i\omega t}) \frac{\partial \theta}{\partial y} = \left(\frac{Pr}{4} \right) \frac{\partial \theta}{\partial t} - (Pr)(Ec) \left(\frac{\partial u}{\partial y} \right)^2 + (Pr)(Q)\theta \quad (9)$$

$$\frac{\partial^2 \phi}{\partial y^2} + (Sc)(1 + \epsilon \alpha e^{i\omega t}) \frac{\partial \phi}{\partial y} = \left(\frac{Sc}{4} \right) \frac{\partial \phi}{\partial t} \quad (10)$$

The corresponding initial and boundary conditions in dimensionless form are:

$$\left. \begin{array}{l} t \leq 0: u = 0, \theta = 0, \phi = 0 \text{ for all } y \\ t > 0: \left\{ \begin{array}{l} u = 0, \theta = T_o(t), \phi = C_o(t) \text{ at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \end{array} \right\} \end{array} \right\} \quad (11)$$

All the physical parameters are defined in the nomenclature.

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin – friction) is given by and in dimensionless form, we obtain Knowing the temperature field, it is interesting to study the effect of the free convection on the rate of heat transfer. This is given by which is written in dimensionless form as

$$\tau = \frac{\tau_w}{\rho u_w^2}, \quad \tau_w = \left[\mu \frac{\partial u}{\partial y} \right]_{y'=0} = \rho v_o^2 u'(0) = \left[\frac{\partial u}{\partial y} \right]_{y=0} \quad (12)$$

The dimensionless local surface heat flux (i.e., Nusselt number) is obtained as

$$Nu = \frac{N_u(x')}{R_{e_x}} = - \left[\frac{\partial \theta}{\partial y} \right]_{y=0} \quad (13)$$

The definition of the local mass flux and the local Sherwood number are respectively given by with the help of these equations, one can write

$$Sh = \frac{S_h(x')}{R_{e_x}} = - \left[\frac{\partial \phi}{\partial y} \right]_{y=0} \quad (14)$$

3 Method of solution

By applying Galerkin finite element method for equations (8)-(10) and Applying the trapezoidal rule, following system of equations in Crank Nicholson method are obtained:

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^n + A_5 u_i^n + A_6 u_{i+1}^n + P^* \quad (15)$$

$$B_1\theta_{i-1}^{n+1} + B_2\theta_i^{n+1} + B_3\theta_{i+1}^{n+1} = B_4\theta_{i-1}^n + B_5\theta_i^n + B_6\theta_{i+1}^n \quad (16)$$

$$D_1C_{i-1}^{n+1} + D_2C_i^{n+1} + D_3C_{i+1}^{n+1} = D_4C_{i-1}^n + D_5C_i^n + D_6C_{i+1}^n \quad (17)$$

Where

$$A_1 = 2 + Nk + 3Brh - 6r, \quad A_2 = 8 + 4Nk + 12r, \quad A_3 = 2 + Nk - 3Brh - 6r,$$

$$A_4 = 2 - Nk - 3Brh + 6r, \quad A_5 = 8 - 4Nk - 12r, \quad A_6 = 2 - Nk + 3Brh + 6r,$$

$$B_1 = 2(\text{Pr}) + 3Brh(\text{Pr}) - 6Zr, \quad B_2 = 8(\text{Pr}) + 12Zr,$$

$$B_3 = 2(\text{Pr}) - 3Brh(\text{Pr}) - 6Zr, \quad B_4 = 2(\text{Pr}) - 3Brh(\text{Pr}) + 6Zr, \quad B_5 = 8(\text{Pr}) - 12Zr,$$

$$B_6 = 2(\text{Pr}) + 3Brh(\text{Pr}) + 6Zr, \quad D_1 = 2(\text{Sc}) + 3Brh(\text{Sc}) - 6r, \quad D_2 = 8(\text{Sc}) + 12r,$$

$$D_3 = 2(\text{Sc}) - 3Brh(\text{Sc}) - 6r, \quad D_4 = 2(\text{Sc}) - 3Brh(\text{Sc}) + 6r, \quad D_5 = 8(\text{Sc}) - 12r,$$

$$D_6 = 2(\text{Sc}) + 3Brh(\text{Sc}) + 6r, \quad P^* = 12Phk = 12hk(\text{Gr})\theta_i^j + 12hk(\text{Gc})C_i^j, \quad Z = 1 + \frac{4}{3R}$$

Here $r = \frac{k}{h^2}$ and h, k are mesh sizes along y -direction and time - direction

respectively. Index i refers to space and j refers to the time. In the equations (15), (16) and (17) taking $i = 1(1)n$ and using boundary conditions (14), then the following system of equations are obtained:

$$A_i X_i = B_i; \quad i = 1(1)n \quad (18)$$

Where A_i 's are matrices of order n and X_i, B_i 's are column matrices having n -components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C-programme. In order to prove the convergence and stability of Galerkin finite element method, the same C-programme was run with smaller values of h and k and no significant change was observed in the values of u, θ and C . Hence the Galerkin finite element method is stable and convergent.

4 Results and Discussions

In order to get a physical insight of the problem, the above physical quantities are computed numerically for different values of the governing parameters

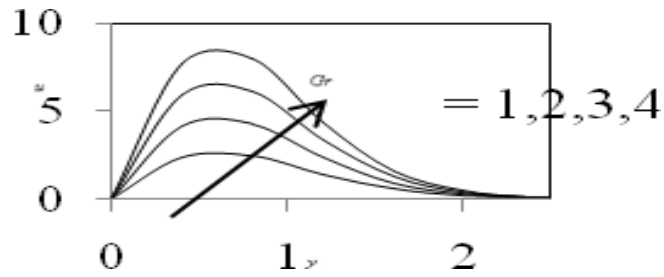


Figure 1. Velocity profiles for different values of Gr

Figure (1) presents the typical velocity profiles in the boundary layer for various values of the thermal Grashof number (Gr). It is observed that an increase in Gr leads to a rise in the values of velocity due to enhancement in buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. In addition, it is observed that the velocity increases sharply near the wall of the porous plate as Gr increases and then decays to the free stream value. For the case of different values of the solutal Grashof number, the velocity profiles in boundary layer are shown in figure (2). The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach a free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increase in the buoyancy force represented by Gc .

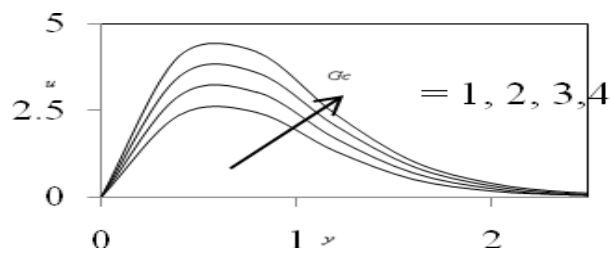


Figure 2. Velocity profiles for different values of Gc

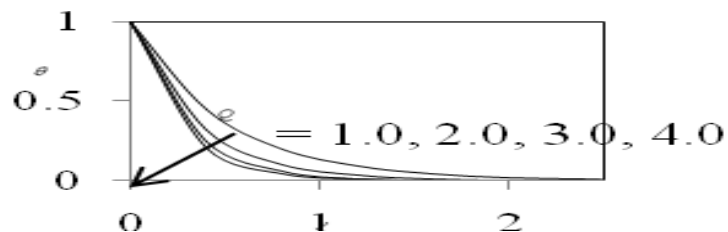


Figure 3. Temperature profiles for different values of Q

Figure (3) illustrate the influence of heat absorption coefficient (Q) on the velocity and temperature at $t = 1.0$ respectively. Physically speaking, the presence of heat absorption (thermal sink) effects has the tendency to reduce the fluid temperature. This causes the thermal buoyancy effects to decrease resulting in a net reduction in the fluid velocity. These behaviors are clearly obvious from figure (3) in which both the velocity and temperature distributions decrease as (Q) increases. It is also observed that the both the hydrodynamic (velocity) and the thermal (temperature) boundary layers decrease as the heat absorption effects increase.

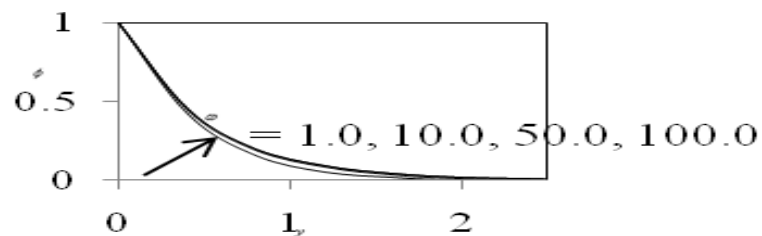


Figure 4. Concentration profiles for different values of ω

5 Conclusions

The governing equations for unsteady MHD convective heat and mass transfer flow past an infinite vertical plate embedded in a porous medium were formulated. Viscous dissipation effects were also included in the present work. The plate velocity is maintained at constant value and the flow was subjected to a transverse magnetic field. The evaluations of the numerical results were performed and graphical results were obtained to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some physical parameters.

1. The Sherwood number (Sh) due to concentration profiles falls under the effect of Schmidt number (Sc) and increases under the effect of Frequency of the suction velocity (ω).

6 Open Problem

This problem has many scientific and engineering applications such as

- (i) Flow of blood through the arteries.
- (ii) Soil mechanics, water purification, and powder metallurgy.
- (iii) Study of the interaction of the geomagnetic field with in the geothermal region.
- (iv) Chemical engineering materials processing, solar porous wafer absorber systems and metallurgy.
- (v) The petroleum engineer concerned with the movement of oil, gas and water through the reservoir of an oil or gas field.

It is hoped that the present work will serve as a vehicle for understanding more complex problems involving the various physical effects investigated in the present problem.

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