

Automated Fuzzy Model Generation and an Analysis of the Proposed Method

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(Communicated by Assoc. Prof. Z. Fuat Toprak)

Abstract

In this paper, software implementation and analysis of a fuzzy modeling technique, which automatically generates the membership functions and fuzzy rules using SMRGT (Simple Membership Functions and Fuzzy Rules Generation Technique), and an application of this product is presented. SMRGT method, introduced by Z. Fuat TOPRAK [10], is a technique used to determine the key values and to generate the rule base of the system. It was introduced in a research article which aimed at fuzzy-modeling of the flow discharge in open canals. In this study, SFM (SMRGT Fuzzy Modeler), a software implementation of automated fuzzy rules generation that incorporates Toprak's methodology, is provided and a few amendments which were previously used in data mining techniques are introduced. Also a brief analysis and critics of the SMRGT method is provided.

Keywords *Fuzzy Modeling – SMRGT – Membership Function and Fuzzy Rule Generation*

1 Introduction

A system model is the mathematical representation of a physical system, a natural system, or an information system. It helps to predict the outputs of the system with the given input variables.

Deterministic systems, which permit the exact prediction of the system outputs, can easily be represented by mathematical models. However, when the deterministic system is too complex, inexact nature of system variables and environmental noise should be put into consideration. Likewise, probabilistic systems, where randomness plays an important role, also becloud the deterministic approaches. Nondeterministic approaches like neural networks, genetic algorithms, fuzzy logic, and such, flourished during recent decades and have a widespread use in system modeling. SMRGT method, studied in this article, uses fuzzy logic approach, which depends on fuzzy set theory adopting uncertainty to reflect human understanding of natural events.

Fuzzy set theory was first mentioned in the article of L. A. Zadeh in 1965 where he defined the notion of fuzzy sets, membership functions, and fuzzy operators such as equality, intersection, union, complement, algebraic operations; and properties of fuzzy sets such as convexity [13]. Fuzzy logic provides an effective means to capture the approximate, inexact nature of the real world [9]. Within a short time it has become the basis for one of the popular modeling methods used in different fields of science, technology and industry.

Zadeh states: In reality, what the theory of fuzzy sets offers is, above all, a much more expressive mathematical language and a much more flexible logic for the characterization and inference from imprecisely defined relations [14]. Modeling natural and artificial events using fuzzy techniques has made a great attraction in that the fuzzy model is closer to human logic and deals with noisy and imprecise data more flexibly than other techniques. This power of fuzzy logic has led to a broad use in technological devices, data analysis, and scientific studies.

Fuzzy models mainly depend on fuzzifying the variables and generating the fuzzy rules. Fuzzifying can be stated as a mapping from crisp values to fuzzy values. Fuzzy values are linguistic variables that are much closer to human understanding. Every fuzzy value has a related membership function that defines the degree of possession of any instance to the same fuzzy value. For instance, when we say "A is tall" we use the fuzzy value, which is the linguistic term "tall" that embraces qualitative meaning. One can also say "A is average in length", if A is also close to the average length. Thus, an instance may both be related to one or more different fuzzy values. This introduces the term Membership Function (MF) that determines the degree of relation of any instance to the fuzzy value. MFs take value in the range 0-1 and can be plotted on XY coordinate system, X-axis being the crisp value of the attribute, and Y-axis being the value of the MF. Shape of an MF depends on the cognition of the expert. Mostly MF diagrams are selected to be piecewise linear, triangular or trapezoidal, since these are simpler, more comprehensible and easier to implement. They also can take the shape of a Gaussian, sigmoid, sinusoid functions, etc.

Membership functions can be determined by the use of an expert's view or can be constructed from available data using different methods such as Genetic

Algorithm, Neural Networks, probability measurement and such [10]. For example, Kissi et al [7], Kim et al.[6], and Chen et al. [3] used Genetic Algorithm to construct Membership functions, H.D. Cheng and J.R. Chen [4] used the maximum entropy principle which is based on information theory to automatically determine the membership functions of brightness of gray levels in digital images in their study.

In a fuzzy model, fuzzy rules determine the fuzzy value of output, given the fuzzy values of inputs. Thus, generation of these rules has an important role in the success of the model. For example the sentence “If A is not tall and if A is overweight then A is heart attack risky” is a fuzzy rule stating A has a risk of heart attack in case A is not tall and if A is overweight. Tallness, overweightness, riskiness embrace fuzziness, thus the rule itself embraces fuzziness.

SMRGT method defines a methodology on how to fuzzify the values by constructing the MFs of the variables and how to generate fuzzy rules. The process is introduced in Toprak (2009) step by step. According to it, dependent and independent variables are first selected, and then minimum and maximum values for each variable are determined. MF shape is then decided. The number of fuzzy values for each variable, which, as Toprak states, must take a value of minimum 3, is then decided. Then key values are determined for the core and the width of the input variables. Since the system modeled was that of a deterministic function $Q=A \times V$ for the demonstration purposes, where Q is the canal flow (m^3/s); A is the cross sectional area (m^2); and V is the cross-sectional average velocity (m/s), the rules table was constructed using the computation of the formula at the key values. In this study the rules are constructed using real data downloaded from UCI Machine Learning repository [2].

This paper is structured as follows: Initially, the fuzzy modeling and SMRGT technique is described in section 2. The implemented program is then introduced and a sample application and related tests are given in section 3. Section 4 includes the results of the sample application using SFM software. Finally in the discussion section a brief discussion on SMRGT method is provided.

2 Methodology

2.1 Qualitative Modeling and Fuzzy Modeling

Human cognition plays an important role in the process of decision-making in most of the systems. It is the role of individuals to manage, operate and take decision over any situation faced. Most of the time, actions to be taken are decided using previous experiences or expert’s knowledge. It is thus necessary to consider the natural way of human understanding and experiences in system modeling. Qualitative modeling based on human cognition has long been used in

social sciences like psychology, sociology, medical science, etc. Qualitative modeling, based on human cognition, is a generalized approach of fuzzy modeling. "...what we mean by a qualitative model is a generalized fuzzy model consisting of linguistic applications about system behavior." [8].

Fuzzy models use fuzzy variables to express linguistic explanations in a more formal way. Fuzzy modeling is based on fuzzy set theory which permits human cognition to have a significant role in system modeling. In Sugeno and Yasukawa (1993) it is simply explained as "...we simply understand the fuzzy modeling to be an approach to form a system model using a description language based on fuzzy logic with fuzzy predicate. In a broader sense we can interpret the fuzzy modeling as a qualitative modeling scheme by which we qualitatively describe system behavior using a natural language. The fuzzy modeling in a narrow sense is a system description with fuzzy quantities".

2.2 Fuzzy Modeling

The main aim of modeling any system is to provide a prediction mechanism for the output of the system in any situation. For non-deterministic models it is impossible to have a hundred percent precision in predicting the output. On the other hand it is impossible to model any non-deterministic system via deterministic methods. Therefore non-deterministic models would be preferred rather than deterministic approaches when a non-deterministic system is being modeled.

In daily life, it is easier for us to give linguistic definitions rather than making exact quantitative assessments. On the other hand, computers use actual values and do not have capability of operating on linguistic terms directly. Fuzzy logic, introduced by Zadeh, makes it easier to handle linguistic values with computers. Modeling any non-deterministic system using fuzzy logic with the help of computers is thus possible.

To model a system with fuzzy logic, first the input and the output variables of the system and fuzzy values of these system variables are determined. MF's are assigned for each of the system variables. Then a set of if-then rules are extracted using data or previous knowledge about the system. A simple fuzzy-rule is given in (1).

If Temperature is Normal and Humidity is Low then Weather is Appropriate (1)

Fuzzy rules consist of two parts, namely antecedent and consequent, and define the outputs for the given values of the input variables. In (1) "Temperature is Normal and Humidity is Low" part is called the antecedent or premise and the "Weather is Appropriate" part is called the consequent. Rule generation is

performed using the data available. In most cases data available is not sufficient or erroneous, thus an intervention of a specialist or an expert, who has knowledge on the system, would be good practice to achieve better models.

2.3 SMRGT Method

SMRGT is a rule-based fuzzy modeling technique. Main question in any given fuzzy system is how to construct the membership functions (MFs) and Fuzzy Rules (FRs), such that the system yields the best results [10]. Thus the technique, Toprak introduces, is a tool to provide a mechanism to construct the MFs and FRs.

Toprak first constructs the MFs for each input variable by dividing the variable ranges into equal pieces. These are the fuzzy values each variable possesses. Toprak, in the definition of SMRGT method, entails the minimum number of MFs for each variable to 3. SFM software, on the other hand, limits the minimum number of fuzzy values for each variable to 2, hence an attribute with only one fuzzy value would be meaningless. The attributes are, thus, split into fuzzy values having isosceles triangular MF diagrams. The first and the last of these triangles are perpendicular triangles with a base half of the inner triangles (Fig. 1 of section 3.2.3). Having fuzzified the variables, FRs are constructed from the data available. Since Toprak uses a theoretical problem, output of any rule can be determined using the results of the given equation. However this approach is impossible for most of the real-life problems. Solution used in our software will be explained in section 3.1.2.

3 The Software

3.1 SFM Software

SFM software is developed using java language on Netbeans 6.9.1 development platform. Software includes modules to read data; model construction, manually intervening fuzzy rules, viewing simple statistics, model testing, saving model and reloading previously saved models.

3.1.1 Input

SFM software reads data from ordinary text files in which data fields are separated by any special separator character. In the data reader screen, input file is selected; number of attributes and separator character is entered. It is thus easier to use any spreadsheet program to preprocess the data file and save it in a text file such as CSV (Comma Separated Values) format.

3.1.2 Modeling

After data is read, modeling modules can be found in the Model Menu. MFs for the variables and rule base are created according to the definition of the SMRGT method as described in [10].

Number of fuzzy values for each variable is entered by the user to be used in the MF creation. However there is also an optimizer that looks for the optimal number of fuzzy values for each variable in a brute force manner.

After the creation of MFs, rule base is constructed using these MF definitions. There are two approaches in SFM software to assign the output values for any rule. First is the 'Nearest Entry' method. In this method, the output class of the data entry that is closest to the rule is assigned to the rule. If the closest data entry does not comprise some of the rule's fuzzy values, then the rule is marked as untrained. The latter method is 'The most existing in the MF Boundaries' which assigns the output class of the most common data entries comprising the fuzzy values of the rule antecedent. In case there is no data entry comprising the rule's fuzzy values, the nearest data entry's output class is assigned to the rule while marked as untrained. These untrained rules as well as any other rule's output class can later be intervened by the user in the program interface.

After the rule list is created and revised by any expert, SFM software permits to reduce the rule list in order to improve the readability and comprehensibility. Suppose we have two input variables, namely weight and length where weight has three –light, normal, heavy- and length has two –short, long- fuzzy values and if the rule list includes the following rules (2) then these two rules can be reduced to (3). This is due to the fact that: regardless of the Length, if the Weight is light then shelter is bad.

If Weight is light and Length is short then shelter is bad
If Weight is light and Length is long then shelter is bad

(2)

If Weight is light then shelter is bad

(3)

SFM software enables to save final model for later use and the saved model can later be reloaded for future use. Also the constructed model can be exported into a Matlab's Mamdani file format to be used by matlab's fuzzy logic library.

3.2 Sample Application

3.2.1 Sample Data

In this study, Vertebral Column Data, supplied in digital format by School of Information and Computer Science, University of California was used in order to

test the SFM Software. It is described in the data repository web page as “a data set containing values for six biomechanical features used to classify orthopaedic patients into 3 classes (normal, disk hernia or spondylolisthesis) or 2 classes (normal or abnormal)” [2].

3.2.2 Preprocessing of the Data

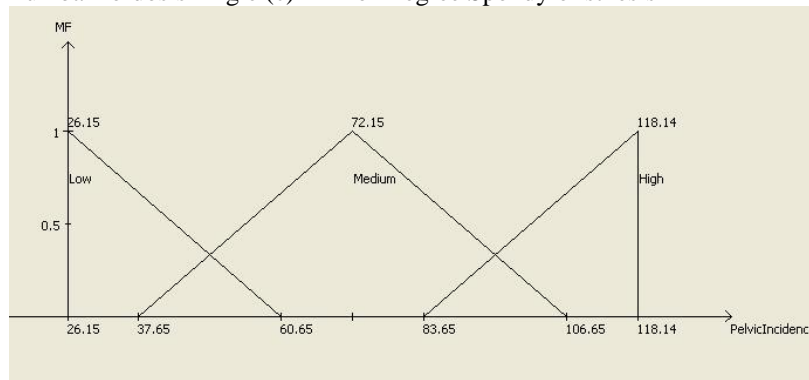
Sample data consists of 310 records, 100 of which are classified as Normal, 60 as Disk Hernia, and 150 as Spondylolisthesis. The class names in the data file were transformed into numerical values 0, 1 and 2 respectively. 116th row having an extreme value of 418.543 for the degree of Spondylolisthesis was deleted.

Original data file consists of 6 attributes, namely Pelvic Incidence (PI), Pelvic Tilt (PT), Lumbar Lordosis Angle (LLA), Sacral Slope (SS), Pelvic Radius (PR), and Degree Spondylolisthesis (DS). These attributes were checked for the correlation with class; and the attributes PI, LLA, and DS having high correlation values were selected for the construction of the model. These preprocessing procedures were applied using a regular spreadsheet program and the output was saved as a text file to be the input file to SFM program.

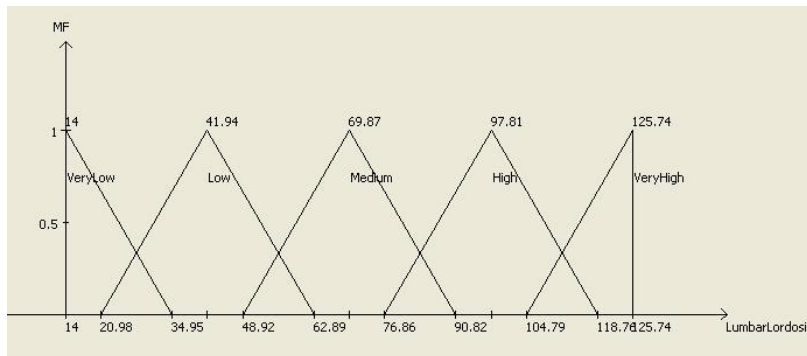
3.2.3 Fuzzy Model of the Vertebral Data

Using SFM software, data file is read and the numbers 3, 5, 4 are assigned for the number of fuzzy values of the attributes PI, LLA, and DS respectively. These numbers were selected by optimizing each attribute’s fuzzy value with maximum value of 5. MF diagrams for these attributes are then captured from the program interface as in Fig 1.

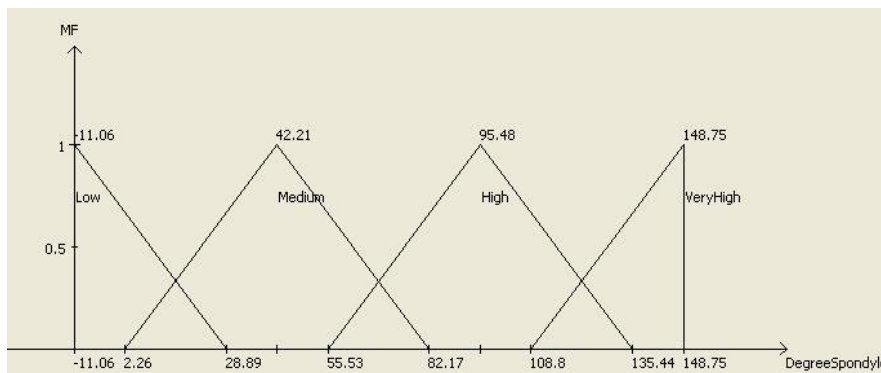
Fig. 1 MF Diagrams for the attributes used in the model. (a) MF for Pelvic Incidence (b) MF for LumbarLordosis Angle (c) MF for Degree Spondylolisthesis



(a)



(b)



(c)

In compliance with the basic principle of counting, $3 \times 5 \times 4 = 60$ fuzzy rules were generated. Class of each rule is then assigned using the most existing class in the boundaries of the fuzzy values on the data. Then, reducing these 60 rules as explained in section 3.1.2, 19 fuzzy rules were generated (Table 1).

Table 1 Reduced Fuzzy Rules List of the Fuzzy Model for Vertebral Column Data

if (DegreeSpondylolisthesis is High) then PatientClass is Spondylolisthesis

if (DegreeSpondylolisthesis is VeryHigh) then PatientClass is Spondylolisthesis

if (PelvicIncidence is Low) and (LumbarLordosis is High) then PatientClass is Spondylolisthesis

if (PelvicIncidence is Low) and (LumbarLordosis is VeryHigh) then PatientClass is Spondylolisthesis

if (PelvicIncidence is Medium) and (LumbarLordosis is High) then PatientClass is Spondylolisthesis

if (PelvicIncidence is High) and (LumbarLordosis is Low) then PatientClass is Spondylolisthesis

if (PelvicIncidence is High) and (DegreeSpondylolisthesis is Medium) then PatientClass is Spondylolisthesis

if (LumbarLordosis is VeryLow) and (DegreeSpondylolisthesis is Low) then PatientClass is DHernia

if (LumbarLordosis is Low) and (DegreeSpondylolisthesis is Medium) then PatientClass is Spondylolisthesis

if (LumbarLordosis is Medium) and (DegreeSpondylolisthesis is Low) then PatientClass is Normal

if (LumbarLordosis is Medium) and (DegreeSpondylolisthesis is Medium) then PatientClass is Spondylolisthesis

if (LumbarLordosis is VeryHigh) and (DegreeSpondylolisthesis is Medium) then PatientClass is Spondylolisthesis

if (PelvicIncidence is Low) and (LumbarLordosis is VeryLow) and (DegreeSpondylolisthesis is Medium)
then PatientClass is DHernia

if (PelvicIncidence is Low) and (LumbarLordosis is Low) and (DegreeSpondylolisthesis is Low)
then PatientClass is Normal

if (PelvicIncidence is Medium) and (LumbarLordosis is VeryLow) and (DegreeSpondylolisthesis is Medium)
then PatientClass is DHernia

if (PelvicIncidence is Medium) and (LumbarLordosis is Low) and (DegreeSpondylolisthesis is Low)
then PatientClass is Normal

if (PelvicIncidence is Medium) and (LumbarLordosis is VeryHigh) and (DegreeSpondylolisthesis is Low)
then PatientClass is Normal

if (PelvicIncidence is High) and (LumbarLordosis is High) and (DegreeSpondylolisthesis is Low)
then PatientClass is Normal

if (PelvicIncidence is High) and (LumbarLordosis is VeryHigh) and (DegreeSpondylolisthesis is Low)
then PatientClass is Normal

3.2.4 Evaluation of the Sample Application

In this particular application a fuzzy model for Vertebral Dysfunction Data is constructed by SMRGT using SFM software. The rules generated consist of trained and untrained rules -highlighted yellow in the SFM program's Edit Rule Interface. Untrained rules were assigned to the class of the incident nearest to the rule's fuzzy values regardless of having the same fuzzy values. These untrained rules should be revised by an expert in a real application, however, in this study, in order to test the accuracy of the technique, this step was skipped. The generated rule list was then tested using the data file itself and an accuracy of 78.64% was achieved.

The attributes were assigned to have 3, 5, 4 fuzzy values corresponding to PI, LLA, and DS respectively. Selection of these numbers depends on the system to be modeled, and the attribute itself. They should be minimal and fit the physical interpretation of the attribute and its categorical distribution. Increasing these numbers would give better accuracy; however it would also mean an increase in the complexity and would possibly result in overfitting of data [12]. As a result of overfitting, accuracy of the models on the training data might not be achieved on real data.

Table 2 lists the accuracy of different models on the same data, constructed by using different number of fuzzy values for the attributes. As can be seen from Table 2, accuracy of the model increases as the assigned number of fuzzy values of the attributes increases. However, the complexity and the overfitting effect should be considered when numbers of fuzzy values (#FV) are selected in real applications. Overfitting effect can easily be observed especially when test data is a separate file that does not include the incidents in the training set. This is illustrated in the results of further tests in section 3.2.5.

Table 2 Accuracy of different models on same data with increasing Number of Fuzzy Values

#FV for PI	#FV for LLA	#FV for DS	Accuracy (%)
2	2	3	70.22
2	3	15	78.96
2	6	15	81.87
15	10	15	84.14

Sample model is constructed using real data; no specialist intervention was applied. In real applications, a specialist revision on rules list would increase the accuracy of the model.

Ajalmar et al. experimented on the same data and achieved slightly higher accuracies [1]. However they have used the two-classified version of the data file namely Normal; and Abnormal Osteophyte Pathology, combining the Disk Hernia and Spondylolisthesis Classes in one. In contrary, model in this study handles Disk Hernia and Spondylolisthesis Classes separately. Also preprocessing of initial file, attribute selection, and use of an expert may have caused accuracy degradation.

3.2.5 Further Tests

The same data was split into two files comprising of 80% for training and 20% for testing purposes. “Nearest Entry” and “Most Existing in the MF Boundaries” methods were applied for Rule Generation Method (RGM). Using the optimizer in SFM software, optimal number of fuzzy values (#FV) for each attribute was determined using the numbers 4, 5, 6, 10 and 15 for the maximum Fuzzy Values. Optimizer uses the training set itself for the calculation of accuracies for its brute force algorithm. Fuzzy models, constructed using these numbers, were tested using both the training set and the test set. Table 3 summarizes the results.

From Table 3, it can be seen that accuracies on training set are significantly higher than accuracies on test sets. It is of the worth to note that even the accuracy on training set seems to improve significantly with the increasing values of the #FV of the attributes, accuracy on test set has no improvement; on the contrary there are even degradations. This is due to overfitting effect. The higher values for #FV is used in the modeling process, the more individual incidence in the training

set is represented in the rule base, thus the accuracy on the training set seems to increase. However this increase will not be the same on other sets such as the test set, since the individuals in the test set are not included in the training set. More generally speaking, using more fuzzy values for the attributes to improve the model will not have the same effect on real data due to overfitting effect. But at the same time the complexity of the model will increase as a side effect.

Table 3 Results for different models using separate training and test sets. (RGM: Rule Generation Method; N.E.: Nearest Entry; ME: Most Existing in the MF Boundaries)

#FV of attributes (PI / LLA / DS)			RGM	Accuracy	
				Training Set (%)	Test Set(%)
2	4	4	N.E.	80.16	75.81
2	5	4	N.E.	80.56	74.19
5	6	4	N.E.	81.38	74.19
6	9	9	N.E.	82.19	72.58
11	11	15	N.E.	86.64	75.81
2	2	4	M.E.	78.54	75.81
3	5	4	M.E.	80.16	74.19
2	6	4	M.E.	80.56	74.19
8	6	10	M.E.	82.59	75.81
12	12	15	M.E.	84.62	75.81

4 Results

SMRGT method, first introduced by Toprak (2009) is a fuzzy modeling technique to extract MFs for system attributes and to generate fuzzy rules for the system. SFM software is the implementation of this method, introduced in this study.

Accuracy of the fuzzy models achieved using SMRGT method is slightly lower than that obtained in [1] where Support Vector Machines (SVM) and Artificial Neural Network (ANN) methods were employed on the same data set and accuracy values between 83.9% - 85.9% were reported. Preprocessing of the data may have great effect on the success of the model so that different models may result in different accuracy results. Another reason of the degradation in accuracy may be the abundance of untrained rules in our sample model which can be dealt with the intervention of a specialist by the use of the SFM software.

Although the algorithm resembles the idea behind the classification algorithms in many ways, the achieved model is composed of fuzzy rules and the variables are reflected in terms of fuzzy variables, thus it is close to human cognition and the result can easily be interpreted and used by human. The developed software, SFM, makes it possible for system experts to intervene and make corrections on fuzzy rules.

Also the SFM software enables to export the constructed model into a Mamdani model that can be processed in Matlab's fuzzy library.

Although a comparative study resulted in superiority of the tests in Ajalmar[1], it is worthy to mention that such comparisons of classification methods are prone to misleading results since the success of any such systems depends on the data, the modeler, preprocessing, parameter selection, and even the intention of the user as stated by Hand [5].

5 Open Problems

One of the handicaps of the SMRGT method is that the ranges of variables are divided into equal pieces (except the first and the last piece being the size of half of the inner pieces) while constructing the corresponding MFs, which is mostly not applicable to real life. For instance one may refer to the range 0°C-10°C as cold, 5°C-40°C as warm and 30°C-100°C as hot, whereas using SMRGT with number of fuzzy values set to 3 for the variable, 0-100°C range would be divided into three pieces with ranges 0°C-37.5°C; 12.5°C-87.5°C; and 62.5°C-100°C. To overcome this handicap, using the SFM software the attributes can be divided into more fuzzy values, at the expense of increasing complexity and unforeseen overfitting effect.

Another issue is the shape of the membership functions. Triangular functions are used in the developed software due to the definition of SMRGT method. This can be extended to include different shapes such as trapezoidal, Gaussian, Sigmoid, and such.

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