Int. J. Open Problems Compt. Math., Vol. 6, No. 4, December, 2013
ISSN 1998-6262;
www.i-csrs.org

On Certain Conditions of Multivariate Power Series Distributions

Sadoon Abdullah Ibrahim Al-Obaidy[1], Faisal Ababneh[2],

S. Alwadi[3], Feras M. Al- Faqih[4]

[1,2,3,4] **Department of Mathematics, Al-Hussein Bin Talal University,**

Ma'an – Jordan

sadoon58@yahoo.com
, Fmababneh@gmail.com
sadam_alwadi@yahoo.co.uk
oxfer@yahoo.com

Abstract

During the last decades, no researches have conducted in order to prove some properties of the of the multivariate power series distribution, as results of the present study proved that any multivariate power series distribution is determined uniquely from the mean –function of any marginal random variable. Furthermore these results indicated also that any given function satisfying certain conditions construct a random vector with multivariate power series distribution which has a mean of the marginal random variable. A useful technique can be applied in model building when we have information about the mean- function.

Key Words: *Multivariate Power Series Distributions, Defining Function, Maclaurin Expansion, Multivariate Logarithmic Distribution, Truncated Power Series Distributions.*

1 Introduction

Many authors deal with multivariate power series distributions and its theories. Korwar (1975), Dehiya and Korwar (1977), Katri (1978a, 1978b), Papageorgiou(1985), Kyriakoussis and Papageorgiou(1989) studied the generalization of characterization by conditional distributions and regression function. Ghosh ,et.al., (1977) considered on his paper a characterization of positive and negative multinomial distributions and some properties of multivariate power series distributions. Rao and Janardan (1982) discussed a general approach to findings the moments of two classes of multivariate discrete distribution. Gupta and Das (2008) derived anew distribution called the quasi multivariate logarithmic series distribution (QMLSD) of order k from the multivariate able series distribution(MASDs) of order k. Simic (2009) calculated the moments of distributions as inflated parameter distribution has been discussed by Momeni (2011). Mahmoudi and Jafari (2012) obtained a new class of distribution contains several lifetime model by compound generalized exponential and power series distributions. Silva, et.al., (2013) introduced a new class of distributions which obtained by compounding the extended Weibull and power series distributions .However, the mentioned researchers have not discussed some properties of multivariate power series, such that the unique and the mean of marginal random variable. Therefore, the present study is an attempt to prove the followings:

Firstly: Any multivariate power series distribution is determined uniquely from the mean-function of any marginal random variable.

Secondly: Any given function satisfying certain conditions constructs a random vector with multivariate power series distribution which has a mean of the marginal random variable. This paper is organized as follows, the first section is the introduction the second section is the methodology then the third section is an empirical example then the forth section is an open problem, then finally is the conclusion.

2 Proposed Method

1. Methodology

Let $f(\theta_1,\theta_2,...,\theta_k)=\sum_{x_1x_2\ldots x_k} a_{x_1x_2\ldots x_k}\theta_1^{x_1}\theta_2^{x_2}\ldots\theta_k^{x_k}$, $a_{x_1x_2\ldots x_k}\geq 0$, be a convergent series for $(\theta_1,\theta_2,...,\theta_k)\in \pi_{i=1}^{k}[0,\mathrm{r}_i]$, r_i is a real number or infinity, the intervals $\left[0,r_i\right]$ are non-intersecting or overlapping, and $x_i = n_i, n_i+1, n_i+2,...$ where n_i, $i=1,2,...,k$ is non-negative integer which maybe the same for all i or different for each I, for more details and discussion refer to (Katri, 1959; Ghosh, et. al., 1977).

Let $p\left(X_1=x_1,X_2=x_2,...,X_k=x_k\right)=\mathrm{a}_{x_1x_2\ldots x_k}\theta_1^{x_1}\theta_2^{x_2}\ldots\theta_k^{x_k}/f\left(\theta_1,\theta_2,...,\theta_k\right)$.

It is easy to see that:

$p\left(X_1=x_1,X_2=x_2,...,X_k=x_k\right)$ is defined if $\theta_i \in \left(0,r_i\right)$, $i=1,2,...,k$. Also

$$\lim_{\theta_i \to 0^+} p\left(X_1 = x_1, X_2 = x_2, ..., X_k = x_k\right) = \lim_{\theta_i \to 0^+} \frac{a_{x_1 x_2 ... x_k} \theta_1^{x_1} \theta_2^{x_2} ... \theta_k^{x_k}}{f(\theta_1, \theta_2, ..., \theta_k)}$$

$$= \lim_{\theta_i \to 0^+} \frac{a_{x_1 x_2 ... x_k} \theta_1^{x_1} \theta_2^{x_2} ... \theta_k^{x_k}}{\sum_{x_i} \psi_{x_i} \theta_i^{x_i}}.$$

Where $\psi_{x_i} = \sum_{x_i ... x_{i-1} x_{i+1} ... x_k} a_{x_1 ... x_{i-1} x_{i+1} ... x_k} \theta_1^{x_1} ... \theta_{i-1}^{x_i -1} \theta_{i+1}^{x_i +1} ... \theta_k^{x_k}$, is independent of θ_i.

Therefore,

$$\lim_{\theta_i \to 0^+} p\left(X_1 = x_1, X_2 = x_2, ..., X_k = x_k\right) = \lim_{\theta_i \to 0^+} \frac{a_{x_1 x_2 ... x_k} \theta_1^{x_1} \theta_2^{x_2} ... \theta_i^{x_i} ... \theta_k^{x_k}}{\psi_{n_i} \theta_i^{n_i} + \psi_{n_i + i} \theta_i^{n_i +1} + ...}$$

$$= \begin{Bmatrix} 1 \text{ if } x_i = n_i, \text{ i= 1, 2, ..., k} \\ 0 \text{ if } x_i \neq n_i \end{Bmatrix}$$

Using this limiting value we see that $P\left(X_1 = x_1, X_2 = x_2, ..., X_k = x_k\right)$ is defined for all x_i and $\theta_i \in (0, r_i)$, **i= 1, 2, ..., k.** the above conditions insure that:

$P\left(X_1 = x_1, X_2 = x_2, ..., X_k = x_k\right) \geq 0$, for all x_i and $\theta_i \in (0, r_i)$.

Also,

$$\sum_{x_1 x_2 ... x_k} P\left(X_1 = x_1, X_2 = x_2, ..., X_k = x_k\right) =$$

$$\frac{1}{f(\theta_1, \theta_2, ..., \theta_k)} \sum_{x_1 x_2 ... x_k} a_{x_1 x_2 ... x_k} \theta_1^{x_1} \theta_2^{x_2} ... \theta_k^{x_k} = 1$$

Because of these properties we say that $(X_1, X_2, ..., X_k)$ is a random vector with multivariate power series distribution in k -parameters.

The function $f(\theta_1, \theta_2, ..., \theta_k)$ called the defining function of the distribution of the random vector $(X_1, X_2, ..., X_k)$.

The mean function of the marginal random variable X_i is :

$$\begin{aligned}
\mu_i(\theta_1,\ \theta_2,\ ...,\ \theta_k) &= \mathrm{EX}_i \\
&= \sum_{x_1 x_2 \dots x_k} x_i P\left(X_1 = x_1,\ X_2 = x_2,\ ...,\ X_k = x_k\right) \\
&= \frac{1}{f(\theta_1,\ \theta_2,\ ...,\ \theta_k)} \sum_{x_1 x_2 \dots x_k} x_i a_{x_1 x_2 \dots x_k}\ \theta_1^{x_1}\ \theta_2^{x_2} \dots \theta_i^{x_i} \dots \theta_k^{x_k} \\
&= \frac{\theta_i}{f(\theta_1,\ \theta_2,\ ...,\ \theta_k)} \sum_{x_1 x_2 \dots x_k} x_i\ a_{x_1 x_2 \dots x_k}\ \theta_1^{x_1}\ \theta_2^{x_2} \dots \theta_i^{x_i - 1}\ \theta_i^{x_i + 1} \dots \theta_k^{x_k} \\
&= \theta_i \frac{\frac{\partial}{\partial \theta_i} f(\theta_1,\ \theta_2,\ ...,\ \theta_k)}{f(\theta_1,\ \theta_2,\ ...,\ \theta_k)} \\
&= \theta_i \frac{\partial}{\partial \theta_i} \ln f(\theta_1,\ \theta_2,\ ...,\ \theta_k)
\end{aligned}$$

It is clear that $\mu_i(\theta_1,\ \theta_2,\ ...,\ \theta_k)$ and $\mu_i(\theta_1,\ \theta_2,\ ...,\ \theta_k)/\theta_i$ are non-negative and continuous functions for all $\theta_i \in \left(0, r_i\right)$.

In fact the two functions are differentiable for all $\theta_i \in \left(0, r_i\right)$. Finally,

$$\begin{aligned}
&\lim_{\theta_i \to 0^+} \mu_i(\theta_1,\ \theta_2,\ ...,\ \theta_k) \\
&= \lim_{\theta_i \to 0^+} \sum_{x_1 x_2 \dots x_k} x_i P\left(X_1 = x_1,\ X_2 = x_2,\ ...,\ X_k = x_k\right) \\
&= \lim_{\theta_i \to 0^+} \frac{\sum_{x_1 x_2 \dots x_k} x_i a_{x_1 x_2 \dots x_k}\ \theta_1^{x_1}\ \theta_2^{x_2} \dots \theta_i^{x_i} \dots \theta_k^{x_k}}{\sum_{\mathrm{X}_i} \psi_{x_i}\ \theta_i^{x_i}} \\
&= \lim_{\theta_i \to 0^+} \frac{\sum_{x_i} x_i\ \psi_{x_i}\ \theta_i^{x_i}}{\sum_{x_i} \psi_{x_i}\ \theta_i^{x_i}}
\end{aligned}$$

$$= \lim_{\theta_i \to 0^+} \frac{n_i\ \psi_{n_i}\ \theta_i^{n_i} + \left(n_i+1\right)\psi_{n_i+1}\ \theta_i^{n_i+1} + \left(n_i+2\right)\psi_{n_i+2}\ \theta_i^{n_i+2} + \dots}{\psi_{n_i}\ \theta_i^{n_i} + \psi_{n_i+1}\ \theta_i^{n_i+1} + \psi_{n_i+2}\ \theta_i^{n_i+2} + \dots}$$

$$= \lim_{\theta_i \to 0^+} \frac{n_i\left(\psi_{n_i}\ \theta_i^{n_i} + \psi_{n_i+1}\ \theta_i^{n_i+1} + \dots\right) + \left(\psi_{n_i+1}\ \theta_i^{n_i+1} + 2\psi_{n_i+2}\ \theta_i^{n_i+2} + \dots\right)}{\psi_{n_i}\ \theta_i^{n_i} + \psi_{n_i+1}\ \theta_i^{n_i+1} + \psi_{n_i+2}\ \theta_i^{n_i+2} + \dots}$$

$$= \lim_{\theta_i \to 0^+} \left\{ n_i + \frac{\theta_i^{n_i+1}\left(\psi_{n_i+1} + 2\psi_{n_i+2}\theta_i + \dots\right)}{\theta_i^{n_i}\left(\psi_{n_i} + \psi_{n_i+1}\theta_i + \psi_{n_i+2}\theta_i^2 + \dots\right)} \right\},\ n_i,\ i=1,2,\dots,k.$$

Theorem1. The multivariate random vector $(X_1, X_2, \dots, X_k)$ having multivariate power series distribution is uniquely determined by the mean of any one marginal random variable of multivariate random vector $(X_1, X_2, \dots, X_k)$.

Proof . Suppose that the mean of the marginal random variable X_i is $m_i(\theta_1, \theta_2, \dots, \theta_k)$, i= 1,2,...,k . Then,

$$m_i(\theta_1, \theta_2, \dots, \theta_k) = \theta_i \frac{\partial}{\partial\theta_i} \ln f(\theta_1, \theta_2, \dots, \theta_k)$$

Let $0 < u \le \theta_i \le t < r_i$. Then for $\theta_i \in (0, r_i)$ we have

$$\frac{m_i(\theta_1, \theta_2, \dots, \theta_k)}{\theta_i} = \frac{\partial}{\partial\theta_i} \ln f(\theta_1, \theta_2, \dots, \theta_k)$$

Therefore, $\displaystyle\int_u^t \frac{m_i(\theta_1, \theta_2, \dots, \theta_k)}{\theta_i}\, d\theta_i = \frac{\partial}{\partial\theta_i}\int_u^t \ln f(\theta_1, \theta_2, \dots, \theta_k)\, d\theta_i$

Implies, $\mathrm{M}_i\left(\theta_1, \theta_2, \dots, \mathrm{t}, \dots, \theta_k\right) - \mathrm{M}_i\left(\theta_1, \theta_2, \dots, \mathrm{u}, \dots, \theta_k\right)$

$$= \ln\left\{\frac{f\left(\theta_1, \theta_2, \dots, \mathrm{t}, \dots, \theta_k\right)}{f\left(\theta_1, \theta_2, \dots, \mathrm{u}, \dots, \theta_k\right)}\right\}$$

Where $\displaystyle\frac{\partial}{\partial\theta_i}\mathrm{M}_i\left(\theta_1, \theta_2, \dots, \theta_k\right) = m_i(\theta_1, \theta_2, \dots, \theta_k)/\theta_i$

Therefore,

$$f\left(\theta_1, \theta_2, \dots, \mathrm{t}, \dots, \theta_k\right) = f\left(\theta_1, \theta_2, \dots, \mathrm{u}, \dots, \theta_k\right)\ e^{-\mathrm{M}_i\left(\theta_1, \theta_2, \dots, \mathrm{u}, \dots, \theta_k\right)}.e^{\mathrm{M}_i\left(\theta_1, \theta_2, \dots, \mathrm{t}, \dots, \theta_k\right)}$$

$$= R\left(\theta_1, \dots, \theta_{i-1}, \mathrm{u}, \theta_{i+1}, \dots, \theta_k\right) e^{\mathrm{M}_i\left(\theta_1, \theta_2, \dots, \mathrm{t}, \dots, \theta_k\right)}$$

Changing t by θ_i we get

$$f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) = R\left(\theta_1,\ ...,\ \theta_{i-1},\ \mathrm{u},\ \theta_{i+1},\ ...,\ \theta_k\right) e^{M_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)} \quad ......\ (1)$$

Since $f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) = \sum_{x_1 x_2 \ldots x_k} a_{x_1 x_2 \ldots x_k}\ \theta_1^{x_1}\ \theta_2^{x_2}\ \ldots \theta_k^{x_k}$

$$= \sum_{x_i} \psi_{x_i}\ \theta_i^{x_i}$$

Therefore, $e^{M_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)}$ has power series in θ_i, say,

$$e^{M_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)} = \sum_{x_i} \psi_{x_i}\ \theta_i^{x_i} \quad \textbf{, Then,}$$

$$P\left(X_i = x_i\right) = \frac{\psi_{x_i}\ \theta_i^{x_i}}{f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)} = \frac{\psi_{x_i}\ \theta_i^{x_i}}{R\left(\theta_1,\ ...,\ \mathrm{u},\ ...,\ \theta_k\right) e^{M_i\left(\theta_1,\ ...,\ \mathrm{t},\ ...,\ \theta_k\right)}}$$

$$= \frac{\psi_{x_i}\ \theta_i^{x_i}}{\sum_{x_i} \psi_{x_i}\ \theta_i^{x_i}}$$

$$= \frac{\psi_{x_i}\ \theta_i^{x_i}}{e^{M_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)}}$$

Hence, without loss of generality we can assume in equation (1) that $R\left(\theta_1,\ ...,\ \mathrm{u},\ ...,\ \theta_k\right) = 1$ and hence equation (1) becomes

$$f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) = e^{M_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)} \quad ....\ (2)$$

To prove uniqueness, suppose that $\left(X_1,\ X_2,\ ...,\ X_k\right)$ and $\left(Y_1,\ Y_2,\ ...,\ Y_k\right)$ are two random vectors having multivariate power series distribution with mean marginal $m_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)$ for both. Assume that the corresponding defining functions are $f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)$ and $g\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)$ respectively. Therefore,

$$\frac{m_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)}{\theta_i} = \frac{\partial}{\partial \theta_i} \ln f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)$$

$$= \frac{\partial}{\partial \theta_i} \ln g\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right), \quad \text{for all } \theta_i \in \left(0, r_i\right)$$

Let $0 < u \le \theta_i \le t < r_i$, then,

$$\int_u^t \frac{\partial}{\partial \theta_i} \ln f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) d\theta_i = \int_u^t \frac{\partial}{\partial \theta_i} \ln g\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) d\theta_i$$

Implies

$$f\left(\theta_1,\ \theta_2,\ \mathbf{t},\ ...,\ \theta_k\right) = \mathbf{H}\left(\theta_1,\ ...,\ \theta_{i-1},\ \mathbf{u},\ \theta_{i+1},\ ...,\ \theta_k\right).\ \mathbf{g}\left(\theta_1,\ ...,\ \mathbf{t},\ ...,\ \theta_k\right)$$

Where $\mathbf{H}\left(\theta_1,\ ...,\ \theta_{i-1},\ \mathbf{u},\ \theta_{i+1},\ ...,\ \theta_k\right) = \dfrac{\mathbf{f}\left(\theta_1,\ ...,\ \mathbf{t},\ ...,\ \theta_k\right)}{\mathbf{g}\left(\theta_1,\ ...,\ \mathbf{t},\ ...,\ \theta_k\right)}$

Changing t by θ_i we get

$$f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) = \mathbf{H}\left(\theta_1,\ ...,\ \theta_{i-1},\ \mathbf{u},\ \theta_{i+1},\ ...,\ \theta_k\right).\ \mathbf{g}\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)\ ....(3)$$

As before and without loss of generality we can assume that:

$\mathbf{H}\left(\theta_1,\ ...,\ \theta_{i-1},\ \mathbf{u},\ \theta_{i+1},\ ...,\ \theta_k\right) = \mathbf{1}$ and hence equation (3) becomes

$$f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) = \mathbf{g}\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right),\ \textbf{for all}\ \ \theta_i\ \in\ \left(\mathbf{0},\boldsymbol{r}_i\right)\ ......\ (4)$$

Therefore from equation (4) it follows that the two random vectors are identical, which completes the proof of the theorem.

Theorem 2. Let $\boldsymbol{m}_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)$ be non-negative and continuous function for all $\theta_i\ \in\ \left(\mathbf{0},\boldsymbol{r}_i\right)$. There exists a multivariate random vector $\left(\boldsymbol{X}_1,\ \boldsymbol{X}_2,\ ...,\boldsymbol{X}_k\right)$ having $\boldsymbol{m}_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)$ as the mean of the marginal random variable $\boldsymbol{X}_i$ iff:

(i) There exist a function $\boldsymbol{M}_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)$ such that

$\dfrac{\partial}{\partial\theta_i}\mathbf{M}_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) = \boldsymbol{m}_i(\theta_1,\ \theta_2,\ ...,\ \theta_k)/\theta_i$, for all $\theta_i\ \in\ \left(\mathbf{0},\boldsymbol{r}_i\right)$,

(ii) $\mathbf{e}^{\mathbf{M}_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)}$ has a Maclaurin expansion in $\theta_1,\ \theta_2,\ ...,\ \theta_k$ with non-negative coefficients.

Proof.

(A). Assume (i) and (ii) hold. Let

$$f\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right) = \mathbf{e}^{\mathbf{M}_i\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)} = \sum_{x_1x_2...x_k} \boldsymbol{a}_{x_1x_2...x_k}\ \theta_1^{x_1}\ \theta_2^{x_2}\ ...\theta_k^{x_k}$$

We can define a multivariate random vector $\left(\boldsymbol{X}_1,\ \boldsymbol{X}_2,\ ...,\boldsymbol{X}_k\right)$ taking values $\left(\boldsymbol{x}_1,\ \boldsymbol{x}_2,\ ...,\boldsymbol{x}_k\right)$ with probability function

$$\boldsymbol{p}\left(\boldsymbol{X}_1 = \boldsymbol{x}_1, \boldsymbol{X}_2 = \boldsymbol{x}_2, ..., \boldsymbol{X}_k = \boldsymbol{x}_k\right) = \boldsymbol{a}_{x_1x_2...x_k}\ \theta_1^{x_1}\ \theta_2^{x_2}...\theta_k^{x_k}\ /\boldsymbol{f}\left(\theta_1,\ \theta_2,\ ...,\ \theta_k\right)$$

Therefore, $\left(X_1, X_2, ..., X_k\right)$ is a random vector with multivariate power series distribution. Furthermore, the mean of the marginal random variable X_i is:

$$\mu_i(\theta_1, \theta_2, ..., \theta_k) = \mathrm{EX}_i = \theta_i \frac{\partial}{\partial\theta_i} \ln f(\theta_1, \theta_2, ..., \theta_k)$$

$$= \theta_i \frac{\partial}{\partial\theta_i} M_i(\theta_1, \theta_2, ..., \theta_k)$$

$$= m_i(\theta_1, \theta_2, ..., \theta_k) \qquad \text{Using } (i)$$

(B) Let $\left(X_1, X_2, ..., X_k\right)$ be a random vector with a multivariate power series distribution and having defining function $f\left(\theta_1, \theta_2, ..., \theta_k\right)$. Then,

$$m_i(\theta_1, \theta_2, ..., \theta_k) = \theta_i \frac{\partial}{\partial\theta_i} \ln f(\theta_1, \theta_2, ..., \theta_k)$$

Implies, $\dfrac{m_i(\theta_1, \theta_2, ..., \theta_k)}{\theta_i} = \dfrac{\partial}{\partial\theta_i} \ln f(\theta_1, \theta_2, ..., \theta_k)$, for all $\theta_i \in \left(0, r_i\right)$.

Let $0 < u \le \theta_i \le t < r_i$. Therefore,

$$\int_u^t \frac{m_i(\theta_1, \theta_2, ..., \theta_k)}{\theta_i} \, d\theta_i = \int_u^t \frac{\partial}{\partial\theta_i} \ln f(\theta_1, \theta_2, ..., \theta_k) \, d\theta_i$$

Implies $f\left(\theta_1, ..., \mathrm{t}, ..., \theta_k\right) = f\left(\theta_1, ..., \mathrm{u}, ..., \theta_k\right) \mathrm{e}^{-\mathrm{M}_i(\theta_1, ..., \mathrm{u}, ..., \theta_k)} . \mathrm{e}^{\mathrm{M}_i(\theta_1, ..., \mathrm{t}, ..., \theta_k)}$

$$= K\left(\theta_1, ..., \theta_{i-1}, \mathrm{u}, \theta_{i+1}, ..., \theta_k\right) \mathrm{e}^{\mathrm{M}_i(\theta_1, \theta_2, ..., \mathrm{t}, ..., \theta_k)}$$

Where $\dfrac{\partial}{\partial\theta_i} \mathrm{M}_i(\theta_1, \theta_2, ..., \theta_k) = m_i(\theta_1, \theta_2, ..., \theta_k)/\theta_i$, $\theta_i \in \left(0, r_i\right)$.

Thus (i) is true, changing t by θ_i we get

$$f\left(\theta_1, \theta_2, ..., \theta_k\right) = K\left(\theta_1, ..., \theta_{i-1}, \mathrm{u}, \theta_{i+1}, ..., \theta_k\right) \mathrm{e}^{\mathrm{M}_i(\theta_1, \theta_2, ..., \theta_k)} \quad ......(5)$$

As before and without loss of generality we can assume in equation (5) that $K\left(\theta_1, ..., \theta_{i-1}, \mathrm{u}, \theta_{i+1}, ..., \theta_k\right) = 1$, and hence equation (5) becomes

$$f\left(\theta_1, \theta_2, ..., \theta_k\right) = \mathrm{e}^{\mathrm{M}_i(\theta_1, \theta_2, ..., \theta_k)} \quad ..... (6)$$

Since $f\left(\theta_1, \theta_2, ..., \theta_k\right)$ has a Maclaurin expansion in $\left(\theta_1, \theta_2, ..., \theta_k\right)$ with non-negative coefficients, then from equation (6) it follows that $\mathrm{e}^{\mathrm{M}_i(\theta_1, \theta_2, ..., \theta_k)}$ has a Maclaurin expansion with the same property. Therefore (ii) is true. This completes the proof of the theorem.

3 Demonstration Example

In order to demonstrate the value of these theories, the following example is given. Example: Let

$$m_i(\theta_1, \theta_2, ..., \theta_k) = -\frac{\theta_i}{(1-\theta_1-\theta_2-...-\theta_k)\ln(1-\theta_1-\theta_2-...-\theta_k)},$$

$0 < \theta_i < 1$, **i=1, 2, ..., k .** Then,

$$\mathrm{M}_i(\theta_1, \theta_2, ..., \theta_k) = \ln\{-A\ln(1-\theta_1-\theta_2-...-\theta_k)\}$$

Where A is independent of θ_i, in general $A = A(\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_k)$

Now $e^{\mathrm{M}_i(\theta_1, \theta_2, ..., \theta_k)} = -A\ln(1-\theta_1-\theta_2-...-\theta_k)$

$$= A\sum_{x_1=1,...x_k=1}\frac{\Gamma(x_1+x_2+...+x_k)}{x_1!x_2!...x_k!}\theta_1^{x_1}\theta_2^{x_2}...\theta_k^{x_k}$$

Therefore, there exists a random vector $(X_1, X_2, ..., X_k)$ having multivariate power series distribution with:

$$m_i(\theta_1, \theta_2, ..., \theta_k) = -\frac{\theta_i}{(1-\theta_1-\theta_2-...-\theta_k)\ln(1-\theta_1-\theta_2-...-\theta_k)}$$

as the mean of the marginal random variable X_i the defining function of this random vector $(X_1, X_2, ..., X_k)$ is:

$$f(\theta_1, \theta_2, ..., \theta_k) = -A\ln(1-\theta_1-\theta_2-...-\theta_k) \quad ...... (7)$$

Without loss of generality we can assume A = 1, and hence equation (7) becomes

$$f(\theta_1, \theta_2, ..., \theta_k) = -\ln(1-\theta_1-\theta_2-...-\theta_k) \quad ...... (8)$$

Therefore

$$p(X_1 = x_1, X_2 = x_2, ..., X_k = x_k) = -\frac{\Gamma(x_1+x_2+...+x_k)\theta_1^{x_1}\theta_2^{x_2}...\theta_k^{x_k}}{x_1!x_2!...x_k!\ln(1-\theta_1-\theta_2-...-\theta_k)}$$

This can be called the multivariate logarithmic distribution.

4 Open Problem and Future Work

As a future work in this area, the researchers can be focus in the following topics:

1- Characterization of polynomial distributions by condition distributions and restricted linear regression.

2- The compound class of general gamma power series distributions.

5 Conclusion

In this study the authors has concluded the results that any multivariate power series distribution is determined uniquely from the mean –function of any marginal random variable. Furthermore these results indicated also that any given function satisfying certain conditions construct a random vector with multivariate power series distribution which has a mean of the marginal random variable.

References

[1] A.Kyriakoussis and H.Papageorgiou, On Characterization of Power Series Distributions by a Marginal Distribution and a Regression Function, Ann. Inst. Statist. Math., 41, No.4 (1989), 671-676.

[2] B.R.Rao and K.G.Janardan , On the Moments of Multivariate Discrete Distributions Using Finite Difference Operators, The American Statistician , 36 (1982),381-383.

[3] C.G.Khatri, On Certain Properties of Power Series Distributions, Biometrika, 46,No.3/4(1959),486-490.

[4] C.G.Khatri, Characterization of Some Discrete Distributions By Linear Regression, J.Indian Statist. Assoc., 16(1978a), 49-58.

[5] C.G.Khatri, Characterization of Some Multivariate Distributions By Conditional Distributions and Linear Regression,J. Indian Statist . ASSOC., 16(1978b), 59-70.

[6] E.Mahmoudi and A.A.Jafari, Generalized Exponential – Power Series Distributions, Computational Statistics and Data Analysis, 56(2012), 4047-4066.

[7] F.Momeni , The Generalized Power Series Distribution and Their Application , The Journal of Mathematics and Computer Science , 2 ,No.4 (2011) , 691-697 .

[8] H.Papageorgiou , On Characterizing Some Bivariate Discrete Distributions , Austral .J. Statist . , 25(1983), 136-144.

[9] H.Papageorgiou , On Characterizing Some Discrete Distributions By a Conditional Distribution and Regression Funtion, Biometrical,J., 27(1985),473-479.

[10] J.K. Ghosh , K.S.Bikas and K.S.Bimal ,Multivariate Power Series Distributions and Neyman's Properties For Multinomials , Journal of Multivariate Analysis , 7 (1977),397-408.

[11] J.Wesolowski , Bivariate Discrete Measures Via a Power Series Conditional Distribution and Regression, Journal Of Multivariate Analysis,55(1995),219-229.
[12] M.K.Patil and D.T.Shirke , Testing Parameter of the Power Series Distribution of a zero-Inflated Power Series Model ,Statistical Methodology , 4,No.4(2007),393-406.
[13] M.K.Patil and D.T.Shirke, Test for Equality of Inflated parameter of Two Zero-Inflated Power Series Distribution, Communications in Statistics, 40, No.14 (2011), 2539-2553.
[14] N.L.Johnson, S.Kotz and A.W.Kemp , Univariate Discrete Distributions , Wiley, New York (1992).
[15] R.B.Silva , M.Bourguignon ,C.R.B. Dias and G.M.Cordeiro, The Compound Class of Extended Weibull Power Series Distributions, Computational Statistics and Data Analysis ,58(2013),352-367.
[16] R.C.Dahiya and R.M.Korwar, On Characterizing Some Bivariate Discrete Distributions By Linear Regression, Sankhya:The Indian Journal of Statistics, 39(1977),124-129.
[17] R.Gupta and K.K.Das, A Family of Multivariate Abel Series Distributions of Order K, Applied Mathematical Sciences, 2, No.45 (2008), 2239-2246.
[18] R.M.Korwar ,ON Characterizing Some Discrete Distributions By Linear Regression, Comm. Statist.,4(1975),1133-1147.
[19] S.R.Patel and V.Vidyanagar, Minimum Variance Unbiased Estimation of Left Truncated Multivariate Modified Power Series Distribution, Metrika, 26(1979), 87-94.
[20] S.Simic, On Moments of the Power Series Distributions, The Open Statistics and Probability Journal, 1(2009), 3-6.