Int. J. Open Problems Compt. Math., Vol. 6, No. 2, June 2013
ISSN 1998-6262; Copyright © ICSRS Publication, 2013
www.i-csrs.org

On Inextensible Flows of Curves According to Type-2 Bishop Frame in E^3

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Abstract

In this paper, we study inextensible flows of curves in Euclidean space $\mathbb{E}^3$***. Using the Frenet frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in Euclidean space*** $\mathbb{E}^3$.

Keywords*: Bienergy, Type-2 Bishop frame, Curvatures, Flows.*

1 Introduction

Construction of fluid flows constitutes an active research field with a high industrial impact. Corresponding real-world measurements in concrete scenarios complement numerical results from direct simulations of the Navier-Stokes equation, particularly in the case of turbulent flows, and for the understanding of the complex spatio-temporal evolution of instationary flow phenomena. More and more advanced imaging devices (lasers, highspeed cameras, control logic, etc.) are currently developed that allow to record fully timeresolved image sequences of fluid flows at high resolutions. As a consequence, there is a need for advanced algorithms for the analysis of such data, to provide the basis for a subsequent pattern analysis, and with abundant applications across various areas, [7,10,11].

In this paper, we study inextensible flows of curves in Euclidean space $\mathbb{E}^3$. Using the Frenet frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in Euclidean space $\mathbb{E}^3$.

2 Preliminaries

Assume that $\{\mathbf{T},\mathbf{N},\mathbf{B}\}$ be the Frenet frame field along α. Then, the Frenet frame satisfies the following Frenet--Serret equations:

$$\begin{aligned}\mathbf{T}' &= \kappa\mathbf{N},\\ \mathbf{N}' &= -\kappa\mathbf{T}+\tau\mathbf{B},\\ \mathbf{B}' &= -\tau\mathbf{N},\end{aligned} \tag{2.1}$$

where κ is the curvature of α and τ its torsion and

$$g(\mathbf{T},\mathbf{T})=1,\ g(\mathbf{N},\mathbf{N})=1,\ g(\mathbf{B},\mathbf{B})=1,$$
$$g(\mathbf{T},\mathbf{N})=g(\mathbf{T},\mathbf{B})=g(\mathbf{N},\mathbf{B})=0.$$

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative, [1]. The Bishop frame is expressed as

$$\begin{aligned}\mathbf{T}' &= k_1\mathbf{M}_1+k_2\mathbf{M}_2,\\ \mathbf{M}_1' &= -k_1\mathbf{T},\\ \mathbf{M}_2' &= -k_2\mathbf{T},\end{aligned} \tag{2.2}$$

where

$$g(\mathbf{T},\mathbf{T})=1,\ g(\mathbf{M}_1,\mathbf{M}_1)=1,\ g(\mathbf{M}_2,\mathbf{M}_2)=1,$$
$$g(\mathbf{T},\mathbf{M}_1)=g(\mathbf{T},\mathbf{M}_2)=g(\mathbf{M}_1,\mathbf{M}_2)=0.$$

Here, we shall call the set $\{\mathbf{T},\mathbf{M}_1,\mathbf{M}_2\}$ as Bishop trihedra, k_1 and k_2 as Bishop curvatures and $\mho(s)=\arctan\frac{k_2}{k_1}$, $\tau(s)=\mho'(s)$ and $\kappa(s)=\sqrt{k_1^2+k_2^2}$.

Bishop curvatures are defined by

$$k_1=\kappa(s)\cos\mho(s),$$
$$k_2=\kappa(s)\sin\mho(s).$$

Let α be a unit speed regular curve and (2.1) be its Frenet--Serret frame. Let us express a relatively parallel adapted frame:

$$\begin{aligned}\mathbf{\Pi}_1' &= -\varepsilon_1\mathbf{B},\\ \mathbf{\Pi}_2' &= -\varepsilon_2\mathbf{B},\\ \mathbf{B}' &= \varepsilon_1\mathbf{\Pi}_1+\varepsilon_2\mathbf{\Pi}_2,\end{aligned} \tag{2.3}$$

where

$$g(\mathbf{B},\mathbf{B})=1,\ g(\mathbf{\Pi}_1,\mathbf{\Pi}_1)=1,\ g(\mathbf{\Pi}_2,\mathbf{\Pi}_2)=1,$$
$$g(\mathbf{B},\mathbf{\Pi}_1)=g(\mathbf{B},\mathbf{\Pi}_2)=g(\mathbf{\Pi}_1,\mathbf{\Pi}_2)=0.$$

We shall call this frame as Type-2 Bishop Frame. In order to investigate this new frame's relation with Frenet--Serret frame, first we write

$$\tau = \sqrt{\varepsilon_1^2 + \varepsilon_2^2}. \quad (2.4)$$

The relation matrix between Frenet--Serret and type-2 Bishop frames can be expressed

$$\mathbf{T} = \sin \mathrm{A}(s)\mathbf{\Pi}_1 - \cos \mathrm{A}(s)\mathbf{\Pi}_2,$$
$$\mathbf{N} = \cos \mathrm{A}(s)\mathbf{\Pi}_1 + \sin \mathrm{A}(s)\mathbf{\Pi}_2,$$
$$\mathbf{B} = \mathbf{B}.$$

So by (2.4), we may express

$$\varepsilon_1 = -\tau \cos \mathrm{A}(s),$$
$$\varepsilon_2 = -\tau \sin \mathrm{A}(s).$$

By this way, we conclude

$$\mathrm{A}(s) = \arctan \frac{\varepsilon_2}{\varepsilon_1}.$$

The frame $\{\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{B}\}$ is properly oriented, and τ and $\mathrm{A}(s) = \int_0^s \kappa(s)ds$ are polar coordinates for the curve α. We shall call the set $\{\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{B}, \boldsymbol{\varepsilon}_1, \varepsilon_2\}$ as type-2 Bishop invariants of the curve α, [17].

3 Inextensible Flows according to New Type-2 Bishop Frame

Let $\alpha(u,t)$ is a one parameter family of smooth curves in E^3.
The arclength of α is given by

$$s(u) = \int_0^u \left| \frac{\partial \alpha}{\partial u} \right| du, \quad (3.1)$$

where

$$\left| \frac{\partial \alpha}{\partial u} \right| = \left| \left\langle \frac{\partial \alpha}{\partial u}, \frac{\partial \alpha}{\partial u} \right\rangle \right|^{\frac{1}{2}}. \quad (3.2)$$

The operator $\frac{\partial}{\partial s}$ is given in terms of u by

$$\frac{\partial}{\partial s} = \frac{1}{v} \frac{\partial}{\partial u},$$

where $v=\left|\frac{\partial\alpha}{\partial u}\right|$ and the arclength parameter is $ds=vdu.$

Any flow of α can be represented as $\{\mathbf{\Pi}_1,\mathbf{\Pi}_2,\mathbf{B}\}$

$$\frac{\partial\alpha}{\partial t}=\mathrm{b}_1\mathbf{\Pi}_1+\mathrm{b}_2\mathbf{\Pi}_2+\mathrm{b}_3\mathbf{B}, \tag{3.3}$$

where $\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3\in C^\infty(\mathrm{E}^3)$.

Definition 3.1. *The flow* $\frac{\partial\alpha}{\partial t}$ *in* E^3 *are said to be inextensible if*

$$\frac{\partial}{\partial t}\left|\frac{\partial\alpha}{\partial u}\right|=0. \tag{3.4}$$

Lemma 3.2. *Let* $\frac{\partial\alpha}{\partial t}=\mathrm{b}_1\mathbf{\Pi}_1+\mathrm{b}_2\mathbf{\Pi}_2+\mathrm{b}_3\mathbf{B}$ *be a smooth flow of the curve* α *according to new type-2 Bishop frame. The flow is inextensible if and only if*

$$\frac{\partial v}{\partial t}=(\frac{\partial\mathrm{b}_1}{\partial u}+\mathrm{b}_3v\varepsilon_1)\sin\mathrm{A}-(\frac{\partial\mathrm{b}_2}{\partial u}+\mathrm{b}_3v\varepsilon_2)\cos\mathrm{A}, \tag{3.5}$$

where $\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3\in C^\infty(\mathrm{E}^3)$.

Proof. Suppose that $\frac{\partial\alpha}{\partial t}$ be a smooth flow of the curve α . Using definition of α, we have

$$\frac{\partial v}{\partial t}=<\sin\mathrm{A}(s)\mathbf{\Pi}_1-\cos\mathrm{A}(s)\mathbf{\Pi}_2,(\frac{\partial\mathrm{b}_1}{\partial u}+\mathrm{b}_3v\varepsilon_1)\mathbf{\Pi}_1$$
$$+(\frac{\partial\mathrm{b}_2}{\partial u}+\mathrm{b}_3v\varepsilon_2)\mathbf{\Pi}_2+(\frac{\partial\mathrm{b}_3}{\partial u}-\mathrm{b}_1v\varepsilon_1-\mathrm{b}_2v\varepsilon_2)\mathbf{B}>.$$

Making necessary calculations from above equation, we have (3.5), which proves the lemma.

Lemma 3.3. *Let* $\frac{\partial\alpha}{\partial t}$ *be a smooth flow of the curve* α *according to new type-2 Bishop frame. The flow is inextensible if and only if*

$$(\frac{\partial\mathrm{b}_1}{\partial u}+\mathrm{b}_3v\varepsilon_1)\sin\mathrm{A}=(\frac{\partial\mathrm{b}_2}{\partial u}+\mathrm{b}_3v\varepsilon_2)\cos\mathrm{A}, \tag{3.6}$$

where $\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3\in C^\infty(\mathrm{E}^3)$.

Proof. Using lemma 3.2, we easily have (3.6).

We now restrict ourselves to arc length parametrized curves. That is, $v=1$ and the local coordinate u corresponds to the curve arc length s. We require the following theorem.

Theorem 3.4.

$$\frac{\partial \mathbf{\Pi}_1}{\partial t}=[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]\mathbf{\Pi}_2+\mathrm{p}_2\mathbf{B},$$
$$\frac{\partial \mathbf{\Pi}_2}{\partial t}=[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]\mathbf{\Pi}_1+\mathrm{p}_4\mathbf{B},$$
$$\frac{\partial \mathbf{B}}{\partial t}=[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]\mathbf{\Pi}_1$$
$$+[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]\mathbf{\Pi}_2,$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6\in C^\infty(\mathrm{E}^3)$.

Proof. Using definition of α, we have

$$\frac{\partial \mathbf{T}}{\partial t}=\frac{\partial}{\partial s}(\mathrm{b}_1\mathbf{\Pi}_1+\mathrm{b}_2\mathbf{\Pi}_2+\mathrm{b}_3\mathbf{B}).$$

Using the (2.3) equations, we have

$$\frac{\partial \mathbf{T}}{\partial t}=(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1)\mathbf{\Pi}_1+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2)\mathbf{\Pi}_2+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\mathbf{B}.$$

Thus it is easy to obtain that

$$\frac{\partial \mathbf{\Pi}_1}{\partial t}=[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]\mathbf{\Pi}_2+\mathrm{p}_2\mathbf{B},$$
$$\frac{\partial \mathbf{\Pi}_2}{\partial t}=[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]\mathbf{\Pi}_1+\mathrm{p}_4\mathbf{B},$$
$$\frac{\partial \mathbf{B}}{\partial t}=[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]\mathbf{\Pi}_1$$
$$+[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]\mathbf{\Pi}_2,$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6\in C^\infty(\mathrm{E}^3)$.

Then, we obtain the theorem.

Theorem 3.5. *Let* $\dfrac{\partial \alpha}{\partial t}$ *be inextensible according to new type-2 Bishop frame. Then, the following partial differential equation holds:*

$$\mathrm{p}_2+\mathrm{p}_5=-[(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}],$$

where $\mathrm{p}_1,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Proof. Assume that $\frac{\partial\alpha}{\partial t}$ is inextensible in three dimensional Euclidean space $\mathbb{E}^3$.

Then we can easily see that

$$\frac{\partial}{\partial s}\frac{\partial}{\partial t}\mathbf{\Pi}_1=\mathrm{p}_2\varepsilon_1\mathbf{\Pi}_1+[\mathrm{p}_2\varepsilon_2+\frac{\partial}{\partial s}[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial\mathrm{A}}{\partial s}\cos\mathrm{A})]]\mathbf{\Pi}_2$$
$$+[\frac{\partial}{\partial s}\mathrm{p}_2-\varepsilon_2[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial\mathrm{A}}{\partial s}\cos\mathrm{A})]]\mathbf{B}.$$

On the other hand, we have

$$\frac{\partial}{\partial t}\frac{\partial}{\partial s}\mathbf{\Pi}_1=-\varepsilon_1[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]\mathbf{\Pi}_1$$
$$-\varepsilon_1[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]\mathbf{\Pi}_2-\frac{\partial\varepsilon_1}{\partial t}\mathbf{B}.$$

Thus, we obtain the theorem.

In the light of Theorem 3.5, we express the following corollary without proof:

Corollary 3.6.

$$\mathrm{p}_2\varepsilon_2+\frac{\partial}{\partial s}[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial\mathrm{A}}{\partial s}\cos\mathrm{A})]]$$
$$=-\varepsilon_1[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Corollary 3.7.

$$-\frac{\partial\varepsilon_1}{\partial t}=[\frac{\partial}{\partial s}\mathrm{p}_2-\varepsilon_2[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial\mathrm{A}}{\partial s}\cos\mathrm{A})]],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Theorem 3.8. *Let* $\frac{\partial\alpha}{\partial t}$ *be inextensible according to new type-2 Bishop frame. Then, the following partial differential equation holds:*

$$-\frac{\partial \varepsilon_2}{\partial t}=[\frac{\partial \mathrm{p}_4}{\partial s}-\varepsilon_1[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin \mathrm{A})\sin \mathrm{A}]],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Proof. Assume that $\frac{\partial \alpha}{\partial t}$ is inextensible in three dimensional Euclidean space E^3.

We can write

$$\frac{\partial}{\partial s}\frac{\partial}{\partial t}\boldsymbol{\Pi}_2=[\mathrm{p}_4\varepsilon_1+\frac{\partial}{\partial s}[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin \mathrm{A})\sin \mathrm{A}]]\,\boldsymbol{\Pi}_1$$
$$+\mathrm{p}_4\varepsilon_2\boldsymbol{\Pi}_2+[\frac{\partial \mathrm{p}_4}{\partial s}-\varepsilon_1[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin \mathrm{A})\sin \mathrm{A}]]\,\mathbf{B}$$

On the other hand, we have

$$\frac{\partial}{\partial t}\frac{\partial}{\partial s}\boldsymbol{\Pi}_2=-\frac{\partial \varepsilon_2}{\partial t}\mathbf{B}-\varepsilon_2[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin \mathrm{A}]\boldsymbol{\Pi}_1$$
$$-\varepsilon_2[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos \mathrm{A}]\,\boldsymbol{\Pi}_2$$

Thus, we obtain the theorem. Hence the proof is completed.

In the light of Theorem 3.8, we express the following corollary without proof:

Corollary 3.9.

$$[\mathrm{p}_4\varepsilon_1+\frac{\partial}{\partial s}[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin \mathrm{A})\sin \mathrm{A}]]$$
$$=-\varepsilon_2[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin \mathrm{A}],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Corollary 3.10.

$$\mathrm{p}_4+\mathrm{p}_6=[(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos \mathrm{A}],$$

where $\mathrm{p}_1,\mathrm{p}_2,\mathrm{p}_3,\mathrm{p}_4,\mathrm{p}_5,\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

Theorem 3.11. *Let* $\frac{\partial \alpha}{\partial t}$ *be inextensible according to new type-2 Bishop frame. Then, the following partial differential equation holds:*

$$\varepsilon_1 \mathrm{p}_2 + \varepsilon_2 \mathrm{p}_4 = -[\varepsilon_1[\mathrm{p}_5 + (\frac{\partial \mathrm{b}_3}{\partial s} - \mathrm{b}_1\varepsilon_1 - \mathrm{b}_2\varepsilon_2)\sin \mathrm{A}]$$
$$+\varepsilon_2[\mathrm{p}_6 - (\frac{\partial \mathrm{b}_3}{\partial s} - \mathrm{b}_1\varepsilon_1 - \mathrm{b}_2\varepsilon_2)\cos \mathrm{A}]]$$

where $\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_3, \mathrm{p}_4, \mathrm{p}_5, \mathrm{p}_6, \mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3$ *are smooth functions of time and arc length.*

Proof. Assume that $\frac{\partial \alpha}{\partial t}$ is inextensible in three dimensional Euclidean space E^3.

Using Theorem 3.4, we have

$$\frac{\partial}{\partial s}\frac{\partial}{\partial t}\mathbf{B} = \frac{\partial}{\partial s}[\mathrm{p}_5 + (\frac{\partial \mathrm{b}_3}{\partial s} - \mathrm{b}_1\varepsilon_1 - \mathrm{b}_2\varepsilon_2)\sin \mathrm{A}]\mathbf{\Pi}_1$$
$$+\frac{\partial}{\partial s}[\mathrm{p}_6 - (\frac{\partial \mathrm{b}_3}{\partial s} - \mathrm{b}_1\varepsilon_1 - \mathrm{b}_2\varepsilon_2)\cos \mathrm{A}]\mathbf{\Pi}_2$$
$$-[\varepsilon_1[\mathrm{p}_5 + (\frac{\partial \mathrm{b}_3}{\partial s} - \mathrm{b}_1\varepsilon_1 - \mathrm{b}_2\varepsilon_2)\sin \mathrm{A}]$$
$$+\varepsilon_2[\mathrm{p}_6 - (\frac{\partial \mathrm{b}_3}{\partial s} - \mathrm{b}_1\varepsilon_1 - \mathrm{b}_2\varepsilon_2)\cos \mathrm{A}]]\mathbf{B}.$$

On the other hand, we have

$$\frac{\partial}{\partial t}\frac{\partial}{\partial s}\mathbf{B} = [\frac{\partial \varepsilon_1}{\partial t} + \varepsilon_2[\mathrm{p}_3 + (\frac{\partial \mathrm{b}_2}{\partial s} + \mathrm{b}_3\varepsilon_2 - \frac{\partial \mathrm{A}}{\partial s}\sin \mathrm{A})\sin \mathrm{A}]]\mathbf{\Pi}_1$$
$$+[\frac{\partial}{\partial t}\varepsilon_2 + \varepsilon_1[\mathrm{p}_1 - \cos \mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s} + \mathrm{b}_3\varepsilon_1 + \frac{\partial \mathrm{A}}{\partial s}\cos \mathrm{A})]]\mathbf{\Pi}_2$$
$$+[\varepsilon_1 \mathrm{p}_2 + \varepsilon_2 \mathrm{p}_4]\mathbf{B}.$$

Thus, we obtain the theorem.

In the light of Theorem 3.11, we express the following corollary without proof:

Corollary 3.12.

$$\frac{\partial}{\partial s}[\mathrm{p}_5 + (\frac{\partial \mathrm{b}_3}{\partial s} - \mathrm{b}_1\varepsilon_1 - \mathrm{b}_2\varepsilon_2)\sin \mathrm{A}] = [\frac{\partial \varepsilon_1}{\partial t} + \varepsilon_2[\mathrm{p}_3 + (\frac{\partial \mathrm{b}_2}{\partial s} + \mathrm{b}_3\varepsilon_2 - \frac{\partial \mathrm{A}}{\partial s}\sin \mathrm{A})\sin \mathrm{A}]],$$

where $\mathrm{p}_3, \mathrm{p}_5, \mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3$ *are smooth functions of time and arc length.*

Corollary 3.13.

$$\frac{\partial}{\partial s}[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos \mathrm{A}]=[\frac{\partial}{\partial t}\varepsilon_2+\varepsilon_1[\mathrm{p}_1-\cos \mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos \mathrm{A})]],$$

where $\mathrm{p}_1\mathrm{p}_6,\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3$ *are smooth functions of time and arc length.*

4 Open Problem

In this work, we study inextensible flows of curves in Euclidean space E^3. We have given some explicit characterizations of curves. Additionally, problems such as; investigation inextensible flows of curves or extending such kind curves to higher dimensional Heisenberg group can be presented as further researches.

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