Int. J. Open Problems Comput. Math. , Vol. 6, No. 1, March, 2013
ISSN 1998-6262; Copyright ©ICSRS Publication, 2013
www.i-csrs.org

On some integral inequalities for s-geometrically convex functions and their applications

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Abstract

In this paper, we establish three inequalities for differentiable s-geometrically and geometrically convex functions which are connected with the famous Hermite-Hadamard inequality holding for convex functions. Some applications to special means of positive real numbers are given.

Keywords: *geometrically convex, s-geometrically convex, hölder inequality, power mean inequality.*

1 Introduction

In this section we will present definitions and some results used in this paper.

Definition 1.1 *Let I be an interval in $\mathbb{R}$. Then $f : I \to \mathbb{R}, \emptyset \neq I \subseteq \mathbb{R}$ is said to be convex if*

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y). \tag{1}$$

for all $x, y \in I$ and $t \in [0, 1]$.

Definition 1.2 *[1] Let $s \in (0, 1]$. A function $f : I \subset \mathbb{R}_0 = [0, \infty) \to \mathbb{R}_0$ is said to be $s-$convex in the second sense if*

$$f(tx + (1-t)y) \leq t^s f(x) + (1-t)^s f(y) \tag{2}$$

for all $x, y \in I$ and $t \in [0, 1]$.

It can be easily checked for $s = 1$, s-convexity reduces to the ordinary convexity of functions defined on $[0, \infty)$.

Recently, In [2], the concept of geometrically and s-geometrically convex functions was introduced as follows.

Definition 1.3 *[2] A function $f : I \subset \mathbb{R}_+ = (0, \infty) \to \mathbb{R}_+$ is said to be a geometrically convex function if*

$$f\left(x^t y^{1-t}\right) \leq [f(x)]^t [f(y)]^{1-t} \tag{3}$$

for all $x, y \in I$ and $t \in [0, 1]$.

Definition 1.4 *[2] A function $f : I \subset \mathbb{R}_+ \to \mathbb{R}_+$ is said to be a s-geometrically convex function if*

$$f\left(x^t y^{1-t}\right) \leq [f(x)]^{t^s} [f(y)]^{(1-t)^s} \tag{4}$$

for some $s \in (0, 1]$, where $x, y \in I$ and $t \in [0, 1]$.

If $s = 1$, the s-geometrically convex function becomes a geometrically convex function on $\mathbb{R}_+$.

Example 1.5 *Let $f(x) = x^s/s$, $x \in (0, 1]$, $0 < s < 1$, $q \geq 1$, and then the function*

$$|f'(x)|^q = x^{(s-1)q} \tag{5}$$

is monotonically decreasing on $(0, 1]$. For $t \in [0, 1]$, we have

$$(s-1)\, q\, (t^s - t) \leq 0, \qquad (s-1)\, q\, ((1-t)^s - (1-t)) \leq 0. \tag{6}$$

Hence, $|f'(x)|^q$ is s-geometrically convex on $(0, 1]$ for $0 < s < 1$.

In [4], the following Lemma and its related Hermite-Hadamard type inequalities for convex functions were obtained.

Lemma 1.6 *[4] Let $f : I^\circ \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a, b \in I^\circ$ with $a < b$. If $f' \in L[a, b]$, then the following equality holds:*

$$\frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)\, dx = \frac{b-a}{2} \int_0^1 (1-2t)\, f'(ta + (1-t)\, b)\, dt. \tag{7}$$

Theorem 1.7 *[4] Let $f : I^\circ \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a, b \in I^\circ$ with $a < b$. If $|f'|$ is convex on $[a, b]$, then the following inequality holds:*

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)\, dx \right| \leq \frac{(b-a)\left(|f'(a)| + |f'(b)|\right)}{8}. \tag{8}$$

Theorem 1.8 *[4] Let $f : I^\circ \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a,b \in I^\circ$ with $a < b$, and let $p > 1$. If the mapping $|f'|^{p/(p-1)}$ is convex on $[a,b]$, then the following inequality holds:*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \tag{9}$$
$$\leq \frac{b-a}{2(p+1)^{1/p}}\left[\frac{|f'(a)|^{p/(p-1)} + |f'(b)|^{p/(p-1)}}{2}\right]^{(p-1)/p}.$$

The goal of this paper is to establish some inequalities of Hermite-Hadamard type for geometrically and s-geometrically convex functions.

2 On some inequalities for s-geometrically convexity

Theorem 2.1 *Let $f : I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$ be a differentiable mapping on I°, $a,b \in I^\circ$ with $a < b$, and $f' \in L[a,b]$. If $|f'|$ is s-geometrically convex and monotonically decreasing on $[a,b]$ for $s \in (0,1]$, then the following inequality holds:*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{b-a}{2}G_1(s; g_1(\alpha), g_2(\alpha)) \tag{10}$$

where

$$g_1(\alpha) = \begin{cases} \frac{1}{4} & \alpha = 1 \\ \frac{2\alpha^{1/2}-2-\ln\alpha}{(\ln\alpha)^2} & \alpha \neq 1 \end{cases}, \quad g_2(\alpha) = \begin{cases} \frac{1}{4} & \alpha = 1 \\ \frac{2\alpha^{1/2}-2\alpha+\alpha\ln\alpha}{(\ln\alpha)^2} & \alpha \neq 1 \end{cases} \tag{11}$$

$$\alpha(u,v) = |f'(a)|^u\,|f'(b)|^{-v},\ u,v > 0,$$

$$G_1(s; g_1(\alpha), g_2(\alpha)) = |f'(b)|^s\left[g_1(\alpha(s,s)) + g_2(\alpha(s,s))\right],\ \ |f'(a)| \leq 1.$$

Proof: Since $|f'|$ is s-geometrically convex and monotonically decreasing on $[a,b]$, from Lemma 1.6, we have

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \\
\leq\ & \left|\frac{b-a}{2}\int_0^1 (1-2t)\,f'(ta+(1-t)\,b)\,dt\right| \\
\leq\ & \frac{b-a}{2}\int_0^1 |1-2t|\,|f'(ta+(1-t)\,b)|\,dt \\
\leq\ & \frac{b-a}{2}\left\{\int_0^{\frac{1}{2}} (1-2t)\left|f'\left(a^t b^{1-t}\right)\right|dt+\int_{\frac{1}{2}}^1 (2t-1)\left|f'\left(a^t b^{1-t}\right)\right|dt\right\} \\
\leq\ & \frac{b-a}{2}\left\{\int_0^{\frac{1}{2}} (1-2t)\,|f'(a)|^{t^s}\,|f'(b)|^{(1-t)^s}\,dt\right. \\
& \left.+\int_{\frac{1}{2}}^1 (2t-1)\,|f'(a)|^{t^s}\,|f'(b)|^{(1-t)^s}\,dt\right\}.
\end{aligned}$$

If $0<\mu\leq 1$, $0<\alpha,s\leq 1$, then

$$\mu^{\alpha^s}\leq \mu^{\alpha s}. \tag{12}$$

If $|f'(a)|\leq 1$, by (12), we get that

$$\begin{aligned}
& \int_0^{\frac{1}{2}} (1-2t)\,|f'(a)|^{t^s}\,|f'(b)|^{(1-t)^s}\,dt+\int_{\frac{1}{2}}^1 (2t-1)\,|f'(a)|^{t^s}\,|f'(b)|^{(1-t)^s}\,dt \\
\leq\ & \int_0^{\frac{1}{2}} (1-2t)\,|f'(a)|^{st}\,|f'(b)|^{s(1-t)}\,dt+\int_{\frac{1}{2}}^1 (2t-1)\,|f'(a)|^{st}\,|f'(b)|^{s(1-t)}\,dt \\
=\ & \int_0^{\frac{1}{2}} (1-2t)\,|f'(b)|^{s}\left|\frac{f'(a)}{f'(b)}\right|^{st}dt+\int_{\frac{1}{2}}^1 (2t-1)\,|f'(b)|^{s}\left|\frac{f'(a)}{f'(b)}\right|^{st}dt \\
=\ & |f'(b)|^{s}\left[g_1(\alpha(s,s))+g_2(\alpha(s,s))\right] \qquad (13)
\end{aligned}$$

Thus, immediately gives the required inequality (10).

Theorem 2.2 *Let $f: I\subseteq\mathbb{R}_+\to\mathbb{R}_+$ be a differentiable mapping on I°, $a,b\in I^\circ$ with $a<b$, and $f'\in L[a,b]$. If $|f'|^q$ is s-geometrically convex and monotonically decreasing on $[a,b]$ for $1/p+1/q=1$ and $s\in(0,1]$, then the following inequality holds:*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right|\leq\frac{b-a}{2(p+1)^{1/p}}G_2(s,q;g_3(\alpha)) \tag{14}$$

where

$$g_3(\alpha) = \begin{cases} 1, & \alpha = 1, \\ \frac{\alpha-1}{\ln \alpha}, & \alpha \neq 1, \end{cases} \tag{15}$$

$$G_2(s,q;g_3(\alpha)) = |f'(b)|^s \left[g_3(\alpha(sq,sq))\right]^{\frac{1}{q}}, \quad |f'(a)| \leq 1. \tag{16}$$

Proof: Since $|f'|^q$ is s-geometrically convex and monotonically decreasing on $[a,b]$, from Lemma 1.6 and Hölder inequality, we have

$$\begin{aligned} & \left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \qquad (17) \\ \leq\ & \frac{b-a}{2}\int_0^1 |1-2t|\,|f'(ta+(1-t)b)|\,dt \\ \leq\ & \frac{b-a}{2}\left(\int_0^1 |1-2t|^p\,dt\right)^{\frac{1}{p}}\left(\int_0^1 |f'(ta+(1-t)b)|^q\,dt\right)^{\frac{1}{q}} \end{aligned}$$

Using the properties of $|f'|^q$, we obtain that

$$\begin{aligned} \left(\int_0^1 |f'(ta+(1-t)b)|^q\,dt\right)^{\frac{1}{q}} &\leq \left(\int_0^1 \left|f'\left(a^t b^{1-t}\right)\right|^q dt\right)^{\frac{1}{q}} \\ &\leq \left(\int_0^1 |f'(a)|^{qt^s}\,|f'(b)|^{q(1-t)^s}\,dt\right)^{\frac{1}{q}}. \qquad (18) \end{aligned}$$

If $|f'(a)| \leq 1$, by (12), we get that

$$\begin{aligned} \left(\int_0^1 |f'(a)|^{qt^s}\,|f'(b)|^{q(1-t)^s}\,dt\right)^{\frac{1}{q}} &\leq \left(\int_0^1 |f'(a)|^{sqt}\,|f'(b)|^{sq(1-t)}\,dt\right)^{\frac{1}{q}} \\ &= \left(|f'(b)|^{sq}\int_0^1 \left|\frac{f'(a)}{f'(b)}\right|^{sqt} dt\right)^{\frac{1}{q}} \\ &= |f'(b)|^s \left[g_3(\alpha(sq,sq))\right]^{\frac{1}{q}}. \qquad (19) \end{aligned}$$

Further, since

$$\int_0^1 |1-2t|^p\,dt = \int_0^{\frac{1}{2}} (1-2t)^p\,dt + \int_{\frac{1}{2}}^1 (2t-1)^p\,dt = 2\int_0^{\frac{1}{2}} (1-2t)^p\,dt = \frac{1}{p+1} \tag{20}$$

a combination of (17)-(20) immediately gives the proof of inequality (14).

Corollary 2.3 *Let $f: I \subseteq (0,\infty) \to (0,\infty)$ be differentiable on I°, $a,b \in I$ with $a < b$, and $f' \in L([a,b])$. If $|f'|^q$ is s-geometrically convex and monotonically decreasing on $[a,b]$ for $s \in (0,1]$, then* *i) When $p=q=2$, one has*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{b-a}{2\sqrt{2}}G_2\left(s,2,g_3(\alpha)\right)$$

ii) If we take $s=1$ in (14), we have for geometrically convex, one has

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{b-a}{2(p+1)^{\frac{1}{p}}}G_2\left(1,q,g_3(\alpha)\right)$$

where g_3, G_2 are same with (15), (16).

Theorem 2.4 *Let $f: I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$ be a differentiable mapping on I°, $a, b \in I^\circ$ with $a < b$, and $f' \in L[a,b]$. If $|f'|^q$ is s-geometrically convex and monotonically decreasing on $[a,b]$ for $q \geq 1$ and $s \in (0,1]$, then the following inequality holds:*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{b-a}{2}\left(\frac{1}{4}\right)^{1-\frac{1}{q}} G_3\left(s,q;g_1(\alpha),g_2(\alpha)\right) \tag{21}$$

where $g_1(\alpha), g_2(\alpha)$ is the same as in (11), and

$$\begin{aligned} & G_3\left(s,q;g_1(\alpha),g_2(\alpha)\right) \\ = \ & |f'(b)|^s\left[\left[g_1\left(\alpha(sq,sq)\right)\right]^{\frac{1}{q}}+\left[g_2\left(\alpha(sq,sq)\right)\right]^{\frac{1}{q}}\right], \quad |f'(a)| \leq 1 \end{aligned}$$

Proof: Since $|f'|^{\,q}$ is s-geometrically convex and monotonically decresing on $[a,b]$, from Lemma 1.6 and well known power mean inequality, we have

$$\begin{aligned} & \left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \\ \leq \ & \frac{b-a}{2}\int_0^1 |1-2t|\,|f'(ta+(1-t)b)|\,dt \\ \leq \ & \frac{b-a}{2}\left[\int_0^{\frac{1}{2}}(1-2t)\,|f'(ta+(1-t)b)|\,dt+\int_{\frac{1}{2}}^1(2t-1)\,|f'(ta+(1-t)b)|\,dt\right] \\ \leq \ & \frac{b-a}{2}\left[\left(\int_0^{\frac{1}{2}}(1-2t)\,dt\right)^{1-\frac{1}{q}}\left[\int_0^{\frac{1}{2}}(1-2t)\,|f'(ta+(1-t)b)|^q\,dt\right]^{\frac{1}{q}}\right. \\ & \left.+\left(\int_{\frac{1}{2}}^1(2t-1)\,dt\right)^{1-\frac{1}{q}}\left[\int_{\frac{1}{2}}^1(2t-1)\,|f'(ta+(1-t)b)|^q\,dt\right]^{\frac{1}{q}}\right] \end{aligned}$$

$$\begin{aligned}
&\leq \frac{b-a}{2}\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\left[\left[\int_0^{\frac{1}{2}}(1-2t)\left|f'\left(a^t b^{1-t}\right)\right|^q dt\right]^{\frac{1}{q}}\right. \\
&\quad \left.+\left[\int_{\frac{1}{2}}^1 (2t-1)\left|f'\left(a^t b^{1-t}\right)\right|^q dt\right]^{\frac{1}{q}}\right] \\
&\leq \frac{b-a}{2}\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\left[\left[\int_0^{\frac{1}{2}}(1-2t)\left|f'(a)\right|^{qt^s}\left|f'(b)\right|^{q(1-t)^s} dt\right]^{\frac{1}{q}}\right. \\
&\quad \left.+\left[\int_{\frac{1}{2}}^1 (2t-1)\left|f'(a)\right|^{qt^s}\left|f'(b)\right|^{q(1-t)^s} dt\right]^{\frac{1}{q}}\right] \qquad (22)
\end{aligned}$$

If $|f'(a)| \leq 1$, by (12), we get that

$$\begin{aligned}
&\int_0^{\frac{1}{2}}(1-2t)\left|f'(a)\right|^{qt^s}\left|f'(b)\right|^{q(1-t)^s} dt \\
\leq\ &\int_0^{\frac{1}{2}}(1-2t)\left|f'(a)\right|^{sqt}\left|f'(b)\right|^{sq(1-t)} dt = \left|f'(b)\right|^{sq} g_1\left(\alpha\left(sq, sq\right)\right), \\
&\int_{\frac{1}{2}}^1 (2t-1)\left|f'(a)\right|^{qt^s}\left|f'(b)\right|^{q(1-t)^s} dt \qquad (23) \\
\leq\ &\int_{\frac{1}{2}}^1 (2t-1)\left|f'(a)\right|^{sqt}\left|f'(b)\right|^{sq(1-t)} dt = \left|f'(b)\right|^{sq} g_2\left(\alpha\left(sq, sq\right)\right)
\end{aligned}$$

By combining of (22)-(23) immediately gives the required inequality (21).

Corollary 2.5 *Let $f : I \subseteq (0,\infty) \to (0,\infty)$ be differentiable on I°, $a, b \in I$ with $a < b$, and $f' \in L([a,b])$. If $|f'|^q$ is s-geometrically convex and monotonically decreasing on $[a,b]$ for $q \geq 1$, and $s \in (0,1]$, then*

i) If we take $q = 1$ in (21), we obtain that

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{b-a}{2} G_3\left(s, 1; g_1(\alpha), g_2(\alpha)\right)$$

ii) If we take $s = 1$ in (21), for geometrically convex, we obtain that

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{b-a}{2}\left(\frac{1}{4}\right)^{1-\frac{1}{q}} G_3\left(1, q; g_1(\alpha), g_2(\alpha)\right)$$

where $g_1(\alpha), g_2(\alpha), \alpha(u,v), G_3(s,q; g_1(\alpha), g_2(\alpha))$ are same with above.

3 Applications to some special means

Let

$$\begin{aligned} A(a,b) &= \frac{a+b}{2},\ L(a,b)=\frac{b-a}{\ln b-\ln a} \qquad (a\neq b),\\ L_p(a,b) &= \left(\frac{b^{p+1}-a^{p+1}}{(p+1)(b-a)}\right)^{1/p},\ a\neq b,\ p\in\mathbb{R},\ p\neq -1,0 \end{aligned}$$

be the arithmetic, logarithmic, generalized logarithmic means for $a,b>0$ respectively.

Proposition 3.1 *Let* $0<a<b\leq 1$, $0<s<1$. *Then*

$$\begin{aligned} & \left|A(a^s,b^s)-[L_s(a,b)]^s\right| \qquad (24)\\ \leq\ & \frac{(b-a)\,sb^{s(s-1)}}{2}L\left(a^{s(s-1)},b^{s(s-1)}\right)\\ & \times\left[A\left(a^{s(s-1)},b^{s(s-1)}\right)-(1/2)\,L\left(a^{s(s-1)},b^{s(s-1)}\right)\right] \end{aligned}$$

Proof: The proof is obvious from Theorem 2.1 applied $f(x)=x^s/s$, $x\in(0,1]$, $0<s<1$. Then $|f'(a)|=a^{s-1}>b^{s-1}=|f'(b)|\geq 1$ and

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right|=\frac{1}{s}\left|A(a^s,b^s)-[L_s(a,b)]^s\right|, \qquad (25)$$

and

$$\begin{aligned} & \left|f'(b)\right|^s\left[g_1(\alpha(s,s))+g_2(\alpha(s,s))\right] \qquad (26)\\ =\ & b^{s(s-1)}\frac{4\sqrt{\left(\frac{a}{b}\right)^{s(s-1)}}-\ln\left(\frac{a}{b}\right)^{s(s-1)}-2\left(\frac{a}{b}\right)^{s(s-1)}+\left(\frac{a}{b}\right)^{s(s-1)}\ln\left(\frac{a}{b}\right)^{s(s-1)}-2}{\left[\ln\left(\frac{a}{b}\right)^{s(s-1)}\right]^2}\\ =\ & \frac{b^{s(s-1)}}{\ln a^{\frac{s-1}{s}}-\ln b^{\frac{s-1}{s}}}\left(\frac{a^{s(s-1)}-b^{s(s-1)}}{b^{s(s-1)}}\right)\left[\frac{a^{s(s-1)}+b^{s(s-1)}}{2b^{s(s-1)}}-\frac{1}{2b^{s(s-1)}}\frac{a^{s(s-1)}-b^{s(s-1)}}{\ln a^{s(s-1)}-\ln b^{s(s-1)}}\right]\\ =\ & b^{s(s-1)}L\left(a^{s(s-1)},b^{s(s-1)}\right)\left[A\left(a^{s(s-1)},b^{s(s-1)}\right)-(1/2)\,L\left(a^{s(s-1)},b^{s(s-1)}\right)\right]. \end{aligned}$$

From (25) and (26), we have the desired inequality.

Proposition 3.2 *Let* $0<a<b\leq 1$, $0<s<1$. *Then*

$$\left|A(a^s,b^s)-[L_s(a,b)]^s\right|\leq\frac{(b-a)\,sb^{sq(1-s)}}{2(p+1)^{1/p}}\left[L\left(a^{sq(s-1)},b^{sq(s-1)}\right)\right]^{1/q} \qquad (27)$$

Proof: The proof is obvious from Theorem 2.2 applied $f(x) = x^s/s$, $x \in (0,1]$, $0 < s < 1$ and $q > 1$. Then $|f'(a)| = a^{s-1} > b^{s-1} = |f'(b)| \geq 1$ and

$$g_3(\alpha(sq, sq)) = \frac{a^{sq(s-1)} - b^{sq(s-1)}}{b^{sq(s-1)}\left(\ln a^{sq(s-1)} - \ln b^{sq(s-1)}\right)} = \frac{1}{b^{sq(s-1)}} L\left(a^{sq(s-1)}, b^{sq(s-1)}\right) \tag{28}$$

From (28), we have the desired inequality.

Proposition 3.3 *Let* $0 < a < b \leq 1$, $0 < s < 1$ *and* $q \geq 1$. *Then*

$$|A(a^s, b^s) - [L_s(a,b)]^s| \leq \frac{s(b-a)}{2}\left(\frac{1}{4}\right)^{1-\frac{1}{q}} b^{s(s-1)}\left[U^{\frac{1}{q}} + V^{\frac{1}{q}}\right] \tag{29}$$

Proof: The proof is obvious from Theorem 2.4 applied $f(x) = x^s/s$, $x \in (0,1]$, $0 < s < 1$ and $q > 1$. Then $|f'(a)| = a^{s-1} > b^{s-1} = |f'(b)| \geq 1$ and

$$g_1(\alpha(sq, sq)) = U = \frac{1}{\ln a^{sq(s-1)} - \ln b^{sq(s-1)}}\left(\frac{1}{b^{\frac{sq(s-1)}{2}}} L\left(a^{\frac{sq(s-1)}{2}}, b^{\frac{sq(s-1)}{2}}\right) - 1\right), \tag{30}$$

$$\begin{aligned} g_2(\alpha(sq, sq)) &= V = \frac{\left(\frac{a}{b}\right)^{2qs(s-1)}}{\left(\ln a^{sq(s-1)} - \ln b^{sq(s-1)}\right)} \times \\ &\left[1 - \frac{\left(\frac{a}{b}\right)^{sq(s-1)} + 1}{\left(\frac{a}{b}\right)^{sq(s-1)}\left(\ln a^{\frac{sq(s-1)}{2}} - \ln b^{\frac{sq(s-1)}{2}}\right)}\right] \end{aligned} \tag{31}$$

From (30) and (31), we have the desired inequality.

4 Open Problem

It is well known that if f is a convex function on the interval $I \subset \mathbb{R}$, then the Hadamard's inequality holds for the convex functions. It has already been proved a lot of this type inequalities for several convex functions. So, there is one questions as follows:

How can be established the general versions of the inequalities (10), (14) and (21) involving several differentiable s-geometrically convex and monotonically decreasing function on I.

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