Int. J. Open Problems Compt. Math., Vol. 5, No. 1, March 2012
ISSN 1998-6262;
www.i-csrs.org

Ordering Policy for Linear Deteriorating Items for Declining Demand with Permissible Delay in Payments

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Abstract

This paper deals with the inventory model for deteriorating items in declining market when delay in payments is allowed to the retailer to settle the account against the purchases made by him. Shortages are not allowed in this model. Here we have dealt with two cases, first one for payment within the permissible time and another for payment after the permissible time. Numerical examples are given to illustrate our results. Sensitivity analysis has been carried out to analyze the changes in the optimal solution with respect to deterioration rate of units in inventory and the rate of change of demand.

Key words: *Deterioration, trade credit, declining demand.*

2000 Mathematics Subject Classification: *90B05*

1. Introduction

In general, the objective of inventory management deals with minimization of the inventory carrying cost. Thus it is very important to determine the optimal stock and optimal time of replenishment of inventory to meet the future demand. This situation becomes more complicated when the inventories are subject to deterioration, delay in payment is permissible and the demand is either increasing or decreasing. An EOQ model under the condition of permissible delay in payments has been developed by Goyal [5] where he has not consider the difference between the selling price and purchase cost. Goyal's model was improved by Dave [4] by assuming the fact that the selling price is higher than its purchase price. Inventory models for the optimal pricing and ordering policies for the retailer under the scenario of allowable trade credit was formulated by Hwang and Shinn [6] and Liao et al. [9]. Considering the difference between the unit sale price and unit purchase cost Jamal et al. [7] and [8] and Sarker et al. [14] have suggested that the retailer should settle the account sooner as the unit selling price increases relative to the unit cost. Most of the above have studied under the assumption of the constant and known deterministic demand. Chang et al. [1] have suggested a model under the condition that supplier offers trade credit to the buyer if the order quantity is greater than or equal to a pre-determined quantity. Further studies in this line are due to Ouyang et al. [10], Chang et al. [2], Chung and Huang [3], Tripathy and Mishra [11 ,12] etc. Teng et al. [15] has suggested the strategy of granting credit items adds not only an additional cost to the supplier but also default risk to the supplier. Ouyang et al. [13] have considered tread credit linked to order quantity for deteriorating items.

In developing the present model demand of a product is assumed to be decreasing function of time. Generally decrease in demand is observed in case of fashionable garments, seasonal products etc. We have considered the case of no shortages and infinite replenishment rate. Here the case of the retailer's generating revenue on unit selling price which is necessarily higher than the unit purchase cost has been considered. We have found the optimal total cost, optimal ordering quantity optimal cycle length for the model. Numerical examples have been given to illustrate the model. Sensitivity analysis has also been carried out to observe the effects on the optimal solution.

2. **Notations and assumptions**

We need the following notations and assumptions to develop the proposed mathematical model.

2.1 Notations

$R(t) = a(1-bt)$: the annual demand as a decreasing function of time where $a > 0$ is fixed demand and $b(0 < b < 1)$ denotes the rate of change of demand.

C : the unit purchase cost.

P : the unit selling price with $(P > C)$.

h : the inventory holding cost per unit per year excluding interest charges.

A : the ordering cost per order.

M : the permissible credit period offered by the supplier to the retailer for settling the account.

I_c : the interest charged per monetary unit in stock per annum by the supplier.

I_e : the interest earned per monetary unit per year, where $I_e < I_c$.

Q : the order quantity.

$\theta = \alpha t$: the linear deterioration rate, where $0 < \alpha << 1$.

$I(t)$: the inventory level at any instant of time t, $0 \le t \le T$.
T : the replenishment cycle time.
$TC(T)$: the total inventory cost per time unit.
Total cost of inventory includes (i) ordering cost, (ii) cost due to deterioration, (iii) inventory holding cost (excluding interest charges), (iv) interest charged on unsold item after the permissible trade credit when $M < T$, and (v) interest earned from sales revenue during the allowable permissible delay in period.

2.2 Assumptions

a. The inventory system under consideration deals with single item.
b. The planning horizon is infinite.
c. The demand of the product is declining function of the time.
d. Shortages are not allowed and lead-time is zero.
e. The deteriorated units can neither be repaired nor replaced during the cycle time.
f. The retailer can deposit generated sales revenue in an interest bearing account during the permissible credit period. At the end of this period, the retailer settles the account for all the units sold keeping the difference for day-to-day expenditure, and paying the interest charges on the unsold items in the stock.

3. Mathematical model

The rate of change of inventory level is governed by the following differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t) \qquad 0 \le t \le T \qquad (1)$$

Subject to the boundary conditions $I(0) = Q$ and $I(T) = 0$.

Since α is very small using series expansion ignoring second and higher powers of α, the solution of (1) will be

$$I(t) = a\left[T - t - \frac{b(T^2 - t^2)}{2} - \frac{\alpha(T^3 - t^3)}{3} + \frac{b\alpha(T^4 - t^4)}{4}\right],\ 0 \le t \le T \quad (2)$$

and the order quantity is

$$Q = a\left[T - \frac{bT^2}{2} - \frac{\alpha T^3}{3} + \frac{b\alpha T^4}{4}\right] \qquad (3)$$

i. Ordering cost; $OC = \frac{A}{T}$ (4)

ii. Cost due to deterioration per unit time;

$$DC = \frac{C}{T}\left[Q - \int_0^T R(t)dt\right] = \frac{C}{T}\left[\frac{ab\alpha T^4}{4} - \frac{a\alpha T^3}{3}\right] \qquad (5)$$

iii. Inventory holding cost per unit time;

$$IHC = \frac{h}{T}\int_0^T I(t)dt = \frac{ah}{T}\left[\frac{T^2}{2} - \frac{bT^3}{3} - \frac{\alpha T^4}{4} - \frac{\alpha bT^4}{4} + \frac{b\alpha T^5}{4}\right] \qquad (6)$$

Here two cases may arise based on the length of T and M using the fact of interest charges or earned (i.e., costs (iv) and (v) in section 2.2),

Case -I: $M < T$

Under the assumption (b) above, the retailer sells $R(M)\ M$ units by the end of the permissible tread credit M and has $CR(M)\ M$ to pay the supplier. The supplier charges an interest rate I_c from time M onwards for the unsold items in the stock. Hence, the interest charged, IC_1 per time unit is

iv. $$IC_1 = \frac{CI_c}{T}\int_M^T I(t)dt$$

$$= \frac{aCI_c}{T}\left[\begin{array}{l} \frac{T^2}{2} - \frac{bT^3}{3} - \frac{\alpha T^4}{4} - \frac{b\alpha T^4}{4} + \frac{b\alpha T^5}{4} - TM + \frac{bT^2 M}{2} \\ -\frac{\alpha T^3 M}{3} + \frac{b\alpha T^4 M}{4} - \frac{M^2}{2} + \frac{bM^3}{6} + \frac{\alpha M^4}{12} - \frac{b\alpha M^4}{4} \end{array}\right] \qquad (7)$$

During $[0, M]$ the retailer sells the product and deposits the revenue into an interest earning account at the rate I_e per monetary unit per year. Since b is very small, using series expansion and ignoring second and higher powers of b, we get the interest earned, IE_1 per time unit

v. $$IE_1 = \frac{PI_e}{T}\int_0^M R(t)tdt = \frac{aPI_e}{T}\left[\frac{M^2}{2} - \frac{bM^3}{3}\right] \qquad (8)$$

Hence, the total cost; $TC_1(T)$ of an inventory system per time unit is

$TC_1(T) = OC + DC + IHC + IC_1 - IE_1$

$$= \left[\begin{array}{l} \frac{A}{T} + \frac{C}{T}\left[\frac{ab\alpha T^4}{4} - \frac{a\alpha T^3}{3}\right] + \frac{ah}{T}\left[\frac{T^2}{2} - \frac{bT^3}{3} - \frac{\alpha T^4}{4} - \frac{\alpha bT^4}{4} + \frac{b\alpha T^5}{4}\right] + \frac{aCI_c}{T} \\ \times \left[\begin{array}{l} \frac{T^2}{2} - \frac{bT^3}{3} - \frac{\alpha T^4}{4} - \frac{b\alpha T^4}{4} + \frac{b\alpha T^5}{4} - TM + \frac{bT^2 M}{2} - \frac{\alpha T^3 M}{3} + \frac{b\alpha T^4 M}{4} \\ -\frac{M^2}{2} + \frac{bM^3}{6} + \frac{\alpha M^4}{12} - \frac{b\alpha M^4}{4} \end{array}\right] - \frac{aPI_e}{T}\left[\frac{M^2}{2} - \frac{bM^3}{3}\right] \end{array}\right]$$

(9)

Case -II: $M \geq T$

Here, the retailer sells *R(T) T*- units in all by the end of the cycle time and has *CR(T) T* to pay the supplier in full by the end of the credit period *M*. Hence, interest charges

iv. $IC_2 = 0$ (10)

and the interest earned per time unit is

v. $$IE_2 = \frac{PI_e}{T}\left[\int_0^T R(t)tdt + R(T)T(M-T)\right]$$

$$= \frac{PI_e}{T}\left[\frac{aT^2}{2} - \frac{abT^3}{3} + aTM - aT^2 - baT^2M + baT^3\right] \quad (11)$$

The total cost; $TC_2(T)$ of an inventory system per time unit is

$$TC_2(T) = OC + DC + IHC + IC_2 - IE_2$$

$$= \left[\begin{array}{l} \frac{A}{T} + \frac{C}{T}\left[\frac{ab\alpha T^4}{4} - \frac{a\alpha T^3}{3}\right] + \frac{ah}{T}\left[\frac{T^2}{2} - \frac{bT^3}{3} - \frac{\alpha T^4}{4} - \frac{b\alpha T^4}{4} + \frac{b\alpha T^5}{4}\right] \\ -\frac{aPI_e}{T}\left[\frac{T^2}{2} - \frac{bT^3}{3} + TM - T^2 - bT^2M + bT^3\right] \end{array}\right]$$

(12)

Hence, the total cost; $TC(T)$ of an inventory system per time unit is

$$TC(T) = \begin{cases} TC_1(T), & M < T \\ TC_2(T), & M \geq T \end{cases} \quad (13)$$

For $T = M$, in equation (12) we have

$$TC_2(M) = \frac{1}{M}\left[\begin{array}{l} A + C\left(\frac{ab\alpha M^4}{4} - \frac{abM^3}{3}\right) + ha\left[\begin{array}{l}\frac{M^2}{2} - \frac{bM^3}{3} - \frac{\alpha M^4}{4} \\ -\frac{b\alpha M^4}{4} + \frac{b\alpha M^5}{4}\end{array}\right] + aCI_e \\ \times\left[\frac{bM^3}{3} - M^2 - \frac{\alpha M^4}{12} - \frac{\alpha bM^4}{2} + \frac{b\alpha M^5}{2}\right] - aPI_e\left[\frac{M^2}{2} - \frac{bM^3}{3}\right] \end{array}\right]$$

(14)

Now $TC_1(T)$ will be minimum, the optimum values of T for the minimum average total cost $TC_1(T)$ is the solution of equation

$$\frac{\partial TC_1(T)}{\partial T} = 0 \quad (15)$$

Provided

$$\frac{\partial^2 TC_1(T)}{\partial T^2} > 0 \quad (16)$$

From equation (15) we get,

$$-\frac{A}{T^2}+C\left[\frac{3ab\alpha T^2}{4}-\frac{2a\alpha T}{3}\right]+ha\left[\frac{1}{2}-\frac{2bT}{3}-\frac{3\alpha T^2}{4}-\frac{3b\alpha T^2}{4}+b\alpha T^3\right]+aCI_c\left[\frac{1}{2}-\frac{2bT}{3}\right.$$

$$\left.-\frac{3\alpha T^2}{4}-\frac{3b\alpha T^2}{4}+b\alpha T^3+\frac{bM}{2}-\frac{2\alpha MT}{3}+\frac{3b\alpha MT^2}{4}\right]+\frac{aPI_e}{T^2}\left[\frac{M^2}{2}-\frac{bM^3}{3}\right]=0$$

(17)

Again, $TC_2(T)$ will be minimum, the optimum values of T for the minimum average total cost $TC_2(T)$ is the solution of equation

$$\frac{\partial TC_2(T)}{\partial T}=0 \tag{18}$$

Provided

$$\frac{\partial^2 TC_2(T)}{\partial T^2}>0 \tag{19}$$

From equation (18) we get,

$$-\frac{A}{T^2}+C\left[\frac{3ab\alpha T^2}{4}-\frac{2a\alpha T}{3}\right]+ha\left[\frac{1}{2}-\frac{2bT}{3}-\frac{3\alpha T^2}{4}-\frac{3b\alpha T^2}{4}+b\alpha T^3\right]$$

$$-aPI_e\left[\frac{4bT}{3}-\frac{1}{2}-bM\right]=0 \tag{20}$$

4. Numerical Examples

Example-1

Let $a=100$ units/year, $b=0.2$, $A=\$100$ per order, $C=\$8$/unit, $P=\$20$/unit, $h=\$60$/unit/annum, $I_c=\$0.12$/year, $I_e=\$0.09$/year, $M=30/365$ years and $\alpha=0.04$/annum in appropriate units. By the help of Mathematica-5.1 software, we obtain the optimum solution for T of Equation (17) of case-I, as $T^*=0.185554$ year which is greater than $M=0.082$ year. Putting T^* in (9) and (3) we get the optimum average cost and ordering quantity as $TC_1(T)^*=1077.20$ and $Q^*=18.2028$ respectively.

Example-2

Let $a=400$ units/year, $b=0.2$, $A=\$100$ per order, $C=\$8$/unit, $P=\$20$/unit, $h=\$60.00$/unit/annum, $I_e=\$0.09$/year, $M=90/365$ years and $\alpha=0.04$/annum in appropriate units. By the help of Mathematica-5.1 software, we obtain the optimum solution for T of Equation (20) of case-II, as $T^*=0.203117$ year which is less than $M=0.246$ year. Putting T^* in (12) and (3) we get the optimum average cost and ordering quantity as $TC_2(T)^*=2758.92$ and $Q^*=79.5532$ respectively.

5. Sensitivity Analysis

We have performed sensitivity analysis by changing parameters b , α and M as 20%, 50%, - 20% and –50% and keeping the remaining parameters at their original values. The corresponding changes in the cycle time, purchase quantities and the total cost are exhibited in table-1 and table-2.

Table-1: Sensitivity analysis for Case -I ($M < T$)

Parameters	% Change	Change in T^*	Change in Q^*	Change in $TVC_1(T)^*$
b	+20	0.186567	18.2306	1074.42
	+50	0.188141	18.2746	1070.17
	-20	0.184569	18.1762	1079.96
	-50	0.183139	18.1381	1084.04
α	+20	0.185627	18.2082	1077.04
	+50	0.185736	18.2162	1076.80
	-20	0.185482	18.1975	1077.39
	-50	0.155374	18.1896	1077.61
M	+20	0.185298	18.1782	1074.07
	+50	0.184828	18.1330	1068.69
	-20	0.185765	18.2231	1079.98
	-50	0.185997	18.2454	1083.48

Table-2: Sensitivity analysis for Case -II ($M > T$)

Parameters	% Change	Change in T^*	Change in Q^*	Change in $TVC_2(T)^*$
b	+20	0.204574	79.7768	2759.59
	+50	0.206806	80.1113	2760.36
	-20	0.201683	79.3289	2758.12
	-50	0.199572	78.9905	2756.69
α	+20	0.203208	79.5794	2758.95
	+50	0.203345	79.6188	2759.00
	-20	0.203026	79.5270	2758.89
	-50	0.202890	79.4878	2758.84
M	+20	0.204657	80.1431	2739.19
	+50	0.206948	81.0203	2709.54
	-20	0.201567	78.9593	2778.63
	-50	0.199222	78.0603	2808.15

6. Results

From table-1, we observed that as rate of change of demand increases, cycle time increases while the average total cost of an inventory system decreases. It is interesting observe that increases in deterioration rate forces retailer to buy more number of units and hence increase cycle time and decrease total cost of an inventory system. Increase in delay period decrease retailer's cycle time and total cost of inventory system.

From table-2, we observed that as rate of change of demand increases, cycle time increases while total cost of an inventory system increase. Increases in deterioration rate forces retailer to buy more number of units and hence inscrease cycle time and total cost of an inventory system. Increases in delay period increase retailer's cycle time and decrease total cost of inventory system.

7. Conclusion

The model developed in this paper assumes demand of a product to be decreasing function of time. Shortages are not allowed and replenishment rate is infinite. It is assumed that the retailer generates revenue on unit selling price which is necessarily higher than the unit purchase cost. The effect of delay period offered by the supplier to retailer is analyzed when the demand of the product is decreasing in the market. The units in inventory are assumed to be subject to time dependent linear deterioration rate. We observe from both cases that increase in credit period 'M' results in the decrease of total inventory cost.

8. Open Problem

The model considered above is suited for items having variable deterioration rate as against earlier models which have considered items having constant rate of deterioration. This model can be used for items like fruits and vegetables whose deterioration rate increases with time. Demand pattern considered here is decreasing function of time, which can also be converted into constant demand pattern. The suggested model can further be extended for fixed credit period with and without shortages. This model can also be further extended for items having quadratic demand or power demand.

Acknowledgements

The authors would like to thank the referees for their valuable comments and suggestions that greatly helped to improve the earlier version of this work to the present state. All of their suggestions were taken care of and incorporated directly in to the text.

References

[1] C. T. Chang, L. Y. Ouyang, and J. T. Teng, "An EOQ model for deteriorating items under supplier credits linked to ordering quantity", *Applied*

Mathematical Modelling, 27, 983 – 996, 2003.

[2] H. J. Chang, J. T. Teng, , L. Y. Ouyang, and C. Y Dye, “Retailer’s optimal pricing and lot-sizing policies for deteriorating items with partial backlogging”, .*European Journal of Operational Research,* 168, 51-64, 2006.

[3] K. J. Chung, and C. K. Huang, “An ordering policy with allowable shortage and permissible delay in payments”, *Applied Mathematical Modelling,* 33, 2518-2525, 2009.

[4] U. Dave, “On Economic order quantity under conditions of permissible delay in payments” *Journal of the Operational Research Society,* 36, 1069, 1985.

[5] S. K. Goyal, “Economic order quantity under conditions of permissible delay in payments”, *Journal of the Operational Research Society,* 36, 335 – 338, 1985.

[6] H. Hwang, and S. W. Shinn, “Retailer’s pricing and lot-sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. *Computers and Operation Research*, 24, 539-547, 1997.

[7] A. M. M. Jamal, B. R. Sarker, and S. Wang, “An ordering policy for deteriorating items with allowable shortages and permissible delay in payment, *Journal of the Operational Research Society*, 48, 826 – 833(1997).

[8] A. M. M. Jamal, B. R. Sarker, and S. Wang, “Optimal payment time for a retailer under permitted delay of payment by the wholesaler”, *International Journal of Production Economics*, 66, 59- 66, 2000.

[9] H. C. Liao, C. H. Tsai, and C.T. Su, “An inventory model with deteriorating items under inflation when a delay in payments is permissible,” *International Journal of Production Economics*, 63, 207 214, 2000.

[10] L. Y. Ouyang, J. T. Teng, and L. H. Chen, “Optimal ordering policy for deteriorating items with partial backlogging under permissible delay in payments”, *Journal of Global Optimization*, 34, 245- 271, 2006.

[11] C. K. Tripathy and U. Mishra, “Ordering policy for Weibull deteriorating items for quadratic demand with permissible delay in payments”, *Applied Mathematical Sciences,* Vol. 4, no. 44, 2181 -2191, 2010.

[12] C. K. Tripathy and U. Mishra, “An EOQ Model for Linear Deteriorating Rates with Shortage and Permissible Delay in Payment” , *The IUP Journal of Operations Management*, Vol. IX, No. 4, pp. 7-20, 2010

[13] L. Y. Ouyang, J. T. Teng, S. K. Goyal, and C. T. Yang, “An economic order quantity model for deteriorating items with partially permissible delay in payments linked to order quantity”, *European Journal of Operational Research,* 194, 418 – 431, 2009.

[14] B. R. Sarker, A. M. M. Jamal, and S. Wang, “Optimal payment time under permissible delay for production with deterioration”, *Production planning and Control*, 11,380 –390, 2000.

[15] J. T. Teng, C.T. Chang, and S. K. Goyal, “Optimal pricing and ordering policy under permissible delay in payments, *International Journal of*

Production Economics; 97, 121 – 129, 2005.

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