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Some Fixed Point Theorems for a Pair of Asymptotically Regular and Compatible Mappings in Fuzzy 2-Metric Space

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Abstract

In this paper we use the concept of a pair of asymptotically regular and compatible mappings to prove some common fixed point theorems in a complete fuzzy 2-metric space.

Keywords: *Compatible mapping; fixed point; fuzzy 2-metric space; Pair of asymptotically regular mapping.*

1 Introduction

Fuzzy set was defined by Zadeh [6]. Kramosil and Michalek[7] introduced fuzzy metric space,George and Veeramani[1] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many researchers have obtained common fixed point theorems for mappings satisfying different types of commutativity conditions. Vasuki [12] proved fixed point theorems for R-weakly commutating mappings. Pant[9,10,11] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems.Balasubramaniam [8],have shown that Rhoades[2] open problem on the existence of contractive definition which generates a fixed point but does not force the mappings to be continuous at the fixed point,posses an affirmative answer. Pant and Jha[11] obtained some analogous results proved by Balasubramanium.

Rhoades [3] introduced the concept of asymptotic regularity for a pair of maps

and Jungck [5] proposed the concept of compatible mappings.
The concept of 2-metric space was initiated by Gahler[14] whose abstract properties were suggested by the area function in Euclidean space. In a paper Sanjay Kumar [13] discussed fuzzy 2-metric space akin to 2-metric spaces introduced by Gahler[14].
This paper presents some common fixed point theorems for a pair of asymptotically regular and compatible mappings in fuzzy 2-metric space.

2 Preliminary Notes

Definition 2.1 [6]A fuzzy set A in X is a function with domain X and values in [0,1].

Definition 2.2 [4]A binary operation $* : [0,1] \times [0,1] \rightarrow [0,1]$*is a continuous t-norms if * is satisfying conditions:*
*1) * is commutative and associative;*
*2) * is continuous;*
3) $a*1 = a \ \ for \ \ all \ \ a \in [0,1]$;
4)$a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *and a,b,c,d*$\in [0,1]$.

Definition 2.3 [1]A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set,* is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, s, t > 0$,
1) M(x,y,t)>0;
2) M(x,y,t)=1 if and only if x=y;
3) M(x,y,t)=M(y,x,t);
4) M(x,y,t)$*M(y,z,s) \leq M(x,z,t+s)$;
5)$M(x,y,.)$*:(0,∞)* $\rightarrow (0,1]$ *is continuous.*
Then M is called a fuzzy metric on X. Then M(x,y,t) denotes the degree of nearness between x and y with respect to t.

Example 2.4 (Induced fuzzy metric [3]) Let (X, d) be a metric space. Denote $a * b = ab$ for all $a, b \in [0,1]$ and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ $defined \ as \ follows:$
$M_d = \frac{t}{t+d(x,y)}$
$Then$(X,$M_d, *$) is a fuzzy metric space.

Definition 2.5 A binary operation $* : [0,1] \times [0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if $([0,1], .)$ is an abelian topological monoid with unit 1 such that $a * b * c \leq d * e * f$ whenever $a \leq d$, $b \leq e$ and $c \leq f$ for all a,b,c,d,e,f$\in [0,1]$.

Definition 2.6 [13]A triplet $(X, M, *)$ is a **fuzzy 2-metric space** if X is an arbitrary set, * is a continuous t-norm, and M is a fuzzy set in $X^3 \times [0, \infty)$ satisfying the following conditions:
1) M(x,y,a,0)=0;
2) $M(x, y, a, t) = 1$ *for all* $t > 0$ *if and only if atleast two of them are equal;*
3) M(x,y,a,t)=M(y,a,x,t)=M(a,y,x,t) (Symmetric);
4) M(x,y,z,r) $*$ *M(x,z,a,s)* $*$ *M(z,y,a,t)* $\leq$ *M(x,y,a,r+s+t) for all x,y,z* $\in X$ *and r,s,t* > 0*;*
5) $M(x, y, a, .) : [0, \infty) \to (0, 1]$ *is left continuous for all x,y,z,a* $\in X$*and* $r, s, t > 0$.
6)$\lim_{n\to\infty} M(x, y, a, t) = 1$ *for all x,y,a* $\in X, t > 0$.

Example 2.7 [13] Let X be the set {1,2,3,4} with 2-metric d defined by,

$$d(x, y, z) = \begin{cases} 0, if x = y, y = z, z = x \ \ and \ \ \{x, y, z\} = \{1, 2, 3\} \\ \frac{1}{2}, otherwise, \end{cases}$$

For each $t \in [0, \infty)$*, define a*$*$*b*$*$*c=abc and*

$$M(x, y, z, t) = \begin{cases} 0, if \ \ t = 0 \\ \frac{t}{t+d(x,y,z)}, if t > 0, where \ \ x, y, z \in X. \end{cases}$$

Then $(X, M, *)$ *is a fuzzy 2-metric space.*

Definition 2.8 [13] (a) A sequence $\{x_n\}$ in $(X, M, *)$ is **Convergent** to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, a, t) = 1$ for each $t > 0$.
(b) A fuzzy 2-metric space, $(X, M, *)$ is called **Cauchy** if $\lim_{n,m\to\infty} M(x_n, x_m, a, t) = 1$ for each $t > 0$.
(c) A fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be **Complete**.

Definition 2.9 Two self mappings f and g of a fuzzy 2-metric space $(X, M, *)$ are called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, a, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 2.10 A sequence $\{x_n\}$ in X is called asymptotically regular with respect to pair (S, T) if $\lim_{n\to\infty} M(Sx_n, Tx_n, a, t) = 1$.

Lemma 2.11 Let $(X, M, *)$ be a fuzzy 2-metric space. If there exists $q \in (0, 1)$ such that $M(x, y, a, qt) \geq M(x, y, a, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.

Proof: If $M(x, y, a, qt) \geq M(x, y, a, t)$,for all $t > 0$ and some constant $0 < q < 1$, then we have, $M(x, y, a, qt) \geq M(x, y, a, \frac{t}{q}) \geq M(x, y, a, \frac{t}{q^2}) \geq$ .... $\geq M(x, y, a, \frac{t}{q^n})$, for all $t > 0$ and $x, y \in X$.Letting $n \to \infty$, we have $M(x, y, a, t) = 1$ and thus $x = y$.

3 Main Results

Theorem 3.1 *Let P,S and T be self mappings of a complete fuzzy 2-metric space* $(X, M, *)$ *with t-norm defined by* $a * b * c = a.b.c$ *where* $a, b, c \in [0, 1]$ *satisfying:*
(i)$M(Px, Py, a, qt) \geq \alpha M(Sy, Py, a, t) + \beta min\{M(Tx, Px, a, t), M(Sx, Px, a, t)$

$$, M(Ty, Py, a, t)\}$$

for all $x, y, a \in X$, *and* $q \in (0, 1)$ *where* $\alpha, \beta > 0, (\alpha + \beta) \geq 1$,
(ii) *the pairs* (P, S) *and* (P, T) *are Compatible,*
(iii)*there exists a sequence* $\{x_n\}$ *which is asymptotically regular with respect to* (P, S) ,(S, T) *and* (P, T),
(iv) *S and T are Continuous,*
Then P,S and T have a unique common fixed point.
Proof: Let $\{x_n\}$ satisfy (iii).From (i), we have
$M(Px_n, Px_m, a, qt) \geq \alpha M(Sx_m, Px_m, a, t) + \beta min\{M(Tx_n, Px_n, a, t),$

$$M(Sx_n, Px_n, a, t), M(Tx_m, Px_m, a, t)\}$$

Making $m, n \to \infty$ and using (iii),we get

$$\geq (\alpha + \beta) \qquad (as, (\alpha + \beta) \geq 1)$$

$$\geq 1.$$

$$\lim_{m,n\to\infty} M(Px_n, Px_m, a, qt) \geq 1.$$

Hence $\{Px_n\}$ is a Cauchy sequence and so converges to some $z \in X$ (as X is complete).
Also,
M(Sx$_n, z, a, r + s + t) \geq M(Sx_n, z, Px_n, r) * M(Sx_n, Px_n, a, s) * M(Px_n, z, a, t)$
Making $n \to \infty$ and using (iii), we have
$\lim_{n\to\infty} M(Sx_n, z, a, r + s + t) \geq 1.$
So,$Sx_n \to z$.Similarly, $Tx_n \to z$.
Now from(iv), we have
$SPx_n \to Sz, S^2x_n = SSx_n \to Sz, STx_n \to Sz.$

$TPx_n \to Tz, T^2x_n = TTx_n \to Tz, TSx_n \to Tz.$
Also, from (ii) we have

$$M(PSx_n, Sz, a, r+s+t) \geq M(PSx_n, Sz, SPx_n, r) * M(PSx_n, SPx_n, a, s)$$

$$*M(SPx_n, Sz, a, t)$$

$$= 1 * 1 * 1$$

$$=1$$

So, $PSx_n \to Sz$. Similarly, $PTx_n \to Tz$. Also from (i) put $x = Sx_n$ and $y = Tx_n$ we get

$$M(PSx_n, PTx_n, a, qt) \geq \alpha M(STx_n, PTx_n, a, t) + \beta min\{M(TSx_n, PSx_n, a, t),$$

$$M(S^2x_n, PSx_n, a, t), M(T^2x_n, PTx_n, a, t)\}$$

Making $n \to \infty$ we get

$$M(Sz, Tz, a, qt) \geq \alpha M(Sz, Tz, a, t) + \beta min\{M(Tz, Sz, a, t),$$

$$M(Sz, Sz, a, t), M(Tz, Tz, a, t)\}$$

$$= (\alpha + \beta)M(Sz, Tz, a, t) \qquad as, (\alpha + \beta) \geq 1)$$

$$\geq M(Sz, Tz, a, t)$$

$\Rightarrow Sz = Tz$. Again from (i) put $x = Tx_n$ and $y = z$ we get

$$M(PTx_n, Pz, a, qt) \geq \alpha M(Sz, Pz, a, t) + \beta min\{M(T^2x_n, PTx_n, a, t)$$

$$, M(STx_n, PTx_n, a, t), M(Tz, Pz, a, t),$$

Making $n \to \infty$ we get
$$M(Tz, Pz, a, qt) \geq \alpha M(Tz, Pz, a, t) + \beta min\{M(Tz, Tz, a, t)$$

$$, M(Sz, Tz, a, t), M(Tz, Pz, a, t)\}$$

$$= (\alpha + \beta)M(Tz, Pz, a, t)$$

$$= M(Tz, Pz, a, t)$$

$$\Rightarrow Tz = Pz = Sz.$$

Also from (i) put $x = Pz$ and $y = z$ we get

$$M(PPz, Pz, a, qt) \geq \alpha M(Sz, Pz, a, t) + \beta min\{M(TPz, P^2z, a, t),$$

$$M(SPz, P^2z, a, t), M(Tz, Pz, a, t)\}$$

$$= \alpha M(Sz, Sz, a, t) + \beta min\{M(TPz, PTz, a, t)$$

$$, M(SPz, PSz, a, t), M(Tz, Tz, a, t)\}$$

$$= (\alpha + \beta) \qquad \text{(From(ii)and } (\alpha + \beta) \geq 1.)$$

$$= 1$$

Hence, $PPz = PSz = Pz = u$ (say).And

$$M(Su, u, a, r + s + t) = M(SPz, u, a, r + s + t)$$

$$\geq M(SPz, PSz, a, s)*M(SPz, PSz, u, r)*M(PSz, u, a, t)$$

$$=1 \qquad \text{(From(ii))}$$

Thus $Su = u$. Similarly $Tu = u$.Thus $Pu = Su = Tu = u$,i.e. u is the common fixed point of P,S and T.
To prove the uniqueness of u, let v be another common fixed point of P,S and T. Then from (i), we have

$$M(Pu, Pv, a, qt) \geq \alpha M(Sv, Pv, a, t) + \beta min\{M(Tu, Pu, a, t)$$

$$\text{,M(Su,Pu,a,t),M(Tv,Pv,a,t)}\}$$

$$\geq (\alpha + \beta) \qquad (as, (\alpha + \beta) \geq 1.)$$

$$\geq 1$$

Hence, $u = v$. This completes the proof of the Theorem (3.1).

Theorem 3.2 *Let P,S and T be self mappings of a complete fuzzy 2-metric space $(X, M, *)$ with t-norm defined by $a * b * c = min\{a, b, c\}$ where $a, b, c \in [0, 1]$ satisfying:*
$(i)M(Px, Py, a, qt) \geq \alpha M(Sy, Py, a, t)+\beta min\{M(Tx, Px, a, t), M(Sx, Px, a, t)$,M(Ty,Py,a,t)$\}$

for all $x, y, a \in X$, *and* $q \in (0,1)$,*where* $\alpha, \beta > 0, (\alpha + \beta) \geq 1$,
(ii)*thepairs(P,S)and(P,T)areCompatible,*
(iii)*thereexistsasequence*$\{x_n\}$ *which is asymptotically regular with respect to* (P, S) *and* (P, T),
(iv) S and T are Continuous,
Then P,S and T have a unique common fixed point.

Proof: The proof of this theorem follows from Theorem 3.1.

Theorem 3.3 *Let P,S and T be self mappings of a complete fuzzy 2-metric space* $(X, M, *)$ *with t-norm defined by* $a * b * c = a.b.c$ *where* $a, b, c \in [0,1]$ *satisfying:*
$(i)M(Px, Py, a, qt) \geq r\{min[M(Sx, Px, a, t), M(Tx, Px, a, t), M(Sy, Py, a, t)$

$$, M(Ty, Py, a, t)]\}$$

for all $x, y, a \in X$, *and* $q \in (0,1)$,*where* $r : [0,1] \to [0,1]$ *is a continuous function such that* $r(t) > t$ *for* $0 \leq t \leq 1$ *and* $r(t) = 1$ *for* $t = 1$,
(ii) the pairs (P, S) *and* (P, T) *are Compatible,*
(iii) there exists a sequence $\{x_n\}$ *which is asymptotically regular with respect to* (P, S) *and* (P, T),
(iv) S and T are Continuous,
Then P,S and T have a unique common fixed point.

Proof: Let $\{x_n\}$ satisfy (iii).From (i), we have
$M(Px_n, Px_m, a, qt) \geq r\{min[M(Sx_n, Px_n, a, t), M(Tx_n, Px_n, a, t), M(Sx_m, Px_m, a, t)$

$$, M(Tx_m, Px_m, a, t)]\}$$

Making $m, n \to \infty$ and using (iii),we get
$\lim_{m,n\to\infty} M(Px_n, Px_m, a, qt) \geq r(1)$.

$$\geq 1.$$

Hence $\{Px_n\}$ is a Cauchy sequence and so converges to some $z \in X$ (as X is complete).
Also,
$M(Sx_n, z, a, r+s+t) \geq M(Sx_n, z, Px_n, r) * M(Sx_n, Px_n, a, s) * M(Px_n, z, a, t)$
Making $n \to \infty$ and using (iii), we have
$\lim_{n\to\infty} M(Sx_n, z, a, r+s+t) \geq 1$.
So,$Sx_n \to z$.Similarly, $Tx_n \to z$.
Now from(iv), we have

$SPx_n \to Sz, S^2x_n = SSx_n \to Sz, STx_n \to Sz.$
$TPx_n \to Tz, T^2x_n = TTx_n \to Tz, TSx_n \to Tz.$
Also, from (ii) we have

$$M(PSx_n, Sz, a, r+s+t) \geq M(PSx_n, Sz, SPx_n, r) * M(PSx_n, SPx_n, a, s)$$
$$*M(SPx_n, Sz, a, t)$$
$$= 1 * 1 * 1$$
$$=1$$

So, $PSx_n \to Sz$. Similarly, $PTx_n \to Tz$. Also from (i) put $x = Sx_n$ and $y = Tx_n$ we get

$$M(PSx_n, PTx_n, a, qt) \geq r\{min[M(S^2x_n, PSx_n, a, t), M(TSx_n, PSx_n, a, t),$$
$$M(STx_n, PTx_n, a, t), M(T^2x_n, PTx_n, a, t)]\}$$

Making $n \to \infty$ we get

$$M(Sz, Tz, a, qt) \geq r\{min[M(Sz, Sz, a, t), M(Tz, Sz, a, t)$$
$$, M(Sz, Tz, a, t), M(Tz, Tz, a, t)]\}$$
$$= r\{min[1, M(Tz, Sz, a, t), M(Sz, Tz, a, t), 1]\}$$
$$= M(Sz, Tz, a, t)$$

$\Rightarrow Sz = Tz$. Again from (i) put $x = Tx_n$ and $y = z$ we get

$$M(PTx_n, Pz, a, qt) \geq r\{min[M(STx_n, PTx_n, a, t), M(T^2x_n$$
$$, PTx_n, a, t), M(Sz, Pz, a, t), M(Tz, Pz, a, t)]\}$$

Making $n \to \infty$ we get
$$M(Tz, Pz, a, qt) \geq r\{min[M(Tz, Tz, a, t), M(Tz, Tz, a, t), M(Tz, Pz, a, t)$$
$$, M(Tz, Pz, a, t)]\}$$
$$= r\{min[1, 1, M(Tz, Pz, a, t), M(Tz, Pz, a, t)]\}$$
$$= M(Tz, Pz, a, t)$$

$$\Rightarrow Tz = Pz = Sz.$$

Also from (i) put $x = Pz$ and $y = z$ we get

$$M(PPz, Pz, a, qt) \geq r\{min[M(SPz, P^2z, a, t), M(TPz, P^2z, a, t)$$

$$, M(Sz, Pz, a, t), M(Tz, Pz, a, t)]\}$$

$$= r\{min[M(SPz, PSz, a, t), M(TPz, PTz, a, t), M(Sz, Sz, a, t),$$

$$M(Pz, Pz, a, t)]\}$$

$$= r\{min[1, 1, 1, 1]\} \qquad \text{(From(ii))}$$

$$= 1$$

Hence, $PPz = PSz = Pz = u$ (say).And

$$M(Su, u, a, r + s + t) = M(SPz, u, a, r + s + t)$$

$$\geq M(SPz, PSz, a, s)*M(SPz, PSz, u, r)*M(PSz, u, a, t)$$

$$=1 \qquad \text{(From(ii))}$$

Thus $Su = u$. Similarly $Tu = u$.Thus $Pu = Su = Tu = u$,i.e. u is the common fixed point of P,S and T.
To prove the uniqueness of u, let v be another common fixed point of P,S and T. Then from (i), we have

$$M(Pu, Pv, a, qt) \geq r\{min[M(Su, Pu, a, t), M(Tu, Pu, a, t), M(Sv, Pv, a, t)$$

$$, M(Tv, Pv, a, t)]\}$$

$$= r\{min[M(Pu, Pu, a, t), M(Pu, Pu, a, t), M(Pv, Pv, a, t)$$

$$, M(Pv, Pv, a, t)]\}$$

$$= r\{min[1, 1, 1, 1]\}$$

$$= 1$$

Hence, $u = v$. This completes the proof of the Theorem (3.1).

Theorem 3.4 *Let P,S and T be self mappings of a complete fuzzy 2-metric space* $(X, M, *)$ *with t-norm defined by* $a * b * c = min\{a, b, c\}$ *where* $a, b, c \in$

$[0,1]$ *satisfying:*
$(i) M(Px, Py, a, qt) \geq r\{min[M(Sx, Px, a, t), M(Tx, Px, a, t), M(Sy, Py, a, t)$

$$, M(Ty, Py, a, t)]\}$$

for all $x, y, a \in X$, *and* $q \in (0,1)$,*where* $r : [0,1] \to [0,1]$ *is a continuous function such that* $r(t) > t$ *for* $0 \leq t \leq 1$ *and* $r(t) = 1$ *for* $t = 1$,
(ii) the pairs (P, S) *and* (P, T) *are Compatible,*
(iii) there exists a sequence $\{x_n\}$ *which is asymptotically regular with respect to* (P, S) *and* (P, T),
(iv) S and T are Continuous,
Then P,S and T have a unique common fixed point.

Proof: The proof follows from Theorem 3.3.

Theorem 3.5 *Let P,S and T be self mappings of a complete fuzzy 2-metric space* $(X, M, *)$ *with t-norm defined by* $a * b * c = a.b.c$ *where* $a, b, c \in [0,1]$ *satisfying:*
$(i) M(Px, Py, a, qt) \geq min\{M(Sx, Px, a, t), M(Tx, Px, a, t), M(Sy, Py, a, t), M(Ty, Py, a, t),$

M(Sy, Ty,a,t)}

for all $x, y, a \in X$, *and* $q \in (0,1)$,
(ii) the pairs (P, S) *and* (P, T) *are Compatible,*
(iii) there exists a sequence $\{x_n\}$ *which is asymptotically regular with respect to* (P, S),(S, T) *and* (P, T),
(iv) S and T are Continuous,
Then P,S and T have a unique common fixed point.

Proof: Proof: Let $\{x_n\}$ satisfy (iii).From (i), we have
$M(Px_n, Px_m, a, qt) \geq min\{M(Sx_n, Px_n, a, t), M(Tx_n, Px_n, a, t), M(Sx_m, Px_m, a, t),$

$$\mathrm{M}(\mathrm{Tx}_m, Px_m, a, t), M(Sx_m, Tx_m, a, t)\}$$

Making $m, n \to \infty$ and using (iii),we get
$\lim_{m,n\to\infty} M(Px_n, Px_m, a, qt) \geq 1$.
Hence $\{Px_n\}$ is a Cauchy sequence and so converges to some $z \in X$ (as X is complete).
Also,
$\mathrm{M}(\mathrm{Sx}_n, z, a, r+s+t) \geq M(Sx_n, z, Px_n, r) * M(Sx_n, Px_n, a, s) * M(Px_n, z, a, t)$
Making $n \to \infty$ and using (iii), we have
$\lim_{n\to\infty} M(Sx_n, z, a, r+s+t) \geq 1$.
So,$Sx_n \to z$.Similarly, $Tx_n \to z$.

Now from(iv), we have
$SPx_n \to Sz, S^2x_n = SSx_n \to Sz, STx_n \to Sz.$
$TPx_n \to Tz, T^2x_n = TTx_n \to Tz, TSx_n \to Tz.$
Also, from (ii) we have

$$M(PSx_n, Sz, a, r+s+t) \geq M(PSx_n, Sz, SPx_n, r) * M(PSx_n, SPx_n, a, s)$$

$$*M(SPx_n, Sz, a, t)$$

$$= 1 * 1 * 1$$

$$=1$$

So, $PSx_n \to Sz$. Similarly, $PTx_n \to Tz$. Also from (i) put $x = Sx_n$ and $y = Tx_n$ we get

$$M(PSx_n, PTx_n, a, qt) \geq min\{M(S^2x_n, PSx_n, a, t), M(TSx_n, PSx_n, a, t),$$

$$M(STx_n, PTx_n, a, t), M(T^2x_n, PTx_n, a, t), M(STx_n, T^2x_n, a, t)\}$$

Making $n \to \infty$ we get

$$M(Sz, Tz, a, qt) \geq min\{M(Sz, Sz, a, t), M(Tz, Sz, a, t), M(Sz, Tz, a, t), M(Tz, Tz, a, t)$$

,M(Sz,Tz,a,t)}

$$= min\{1, M(Tz, Sz, a, t), M(Sz, Tz, a, t), 1, M(Sz, Tz, a, t)\}$$

$$= M(Sz, Tz, a, t)$$

$\Rightarrow Sz = Tz$. Again from (i) put $x = Tx_n$ and $y = z$ we get

$$M(PTx_n, Pz, a, qt) \geq min\{M(STx_n, PTx_n, a, t), M(T^2x_n, PTx_n, a, t), M(Sz, Pz, a, t),$$

$$M(Tz, Pz, a, t), M(Sz, Tz, a, t)\}$$

Making $n \to \infty$ we get

$$M(Tz, Pz, a, qt) \geq min\{M(Tz, Tz, a, t), M(Tz, Tz, a, t), M(Tz, Pz, a, t), M(Tz, Pz, a, t)$$

,M(Tz,Tz,a,t)}

$$= min\{1, 1, M(Tz, Pz, a, t), M(Tz, Pz, a, t), 1\}$$

$$= M(Tz, Pz, a, t)$$

$$\Rightarrow Tz = Pz = Sz.$$

Also from (i) put $x = Pz$ and $y = z$ we get

$$M(PPz, Pz, a, qt) \geq min\{M(SPz, P^2z, a, t), M(TPz, P^2z, a, t), M(Sz, Pz, a, t),$$

$$M(Tz, Pz, a, t), M(Sz, Tz, a, t)\}$$

$$= min\{M(SPz, PSz, a, t), M(TPz, PTz, a, t), M(Sz, Sz, a, t),$$

$$M(Pz, Pz, a, t), M(Sz, Sz, a, t)\}$$

$$= min\{1, 1, 1, 1, 1\} \qquad \text{(From(ii))}$$

$$= 1$$

Hence, $PPz = PSz = Pz = u$ (say).And

$$M(Su, u, a, r + s + t) = M(SPz, u, a, r + s + t)$$

$$\geq M(SPz, PSz, a, s)*M(SPz, PSz, u, r)*M(PSz, u, a, t)$$

$$=1 \qquad \text{(From(ii))}$$

Thus $Su = u$. Similarly $Tu = u$.Thus $Pu = Su = Tu = u$,i.e. u is the common fixed point of P,S and T.
To prove the uniqueness of u, let v be another common fixed point of P,S and T. Then from (i), we have

$$M(Pu, Pv, a, qt) \geq min\{M(Su, Pu, a, t), M(Tu, Pu, a, t), M(Sv, Pv, a, t), M(Tv, Pv, a, t)$$

$$,\mathrm{M(Sv,Tv,a,t)}\}$$

$$= min\{M(Pu, Pu, a, t), M(Pu, Pu, a, t), M(Pv, Pv, a, t), M(Pv, Pv, a, t)$$

$$,\mathrm{M(Pv,Pv,a,t)}\}$$

$$= min\{1, 1, 1, 1, 1\}$$

$$= 1$$

Hence, $u = v$. This completes the proof of the Theorem (3.1).

Theorem 3.6 *Let P,S and T be self mappings of a complete fuzzy 2-metric space* $(X, M, *)$ *with t-norm defined by* $a * b * c = min\{a, b, c\}$ *where* $a, b, c \in$

$[0, 1]$ *satisfying:*
$(i) M(Px, Py, a, qt) \geq min\{M(Sx, Px, a, t), M(Tx, Px, a, t), M(Sy, Py, a, t), M(Ty, Py, a, t)$

$, M(Sy, Ty, a, t)\}$

for all $x, y, a \in X$, *and* $q \in (0, 1)$,
(ii) the pairs (P, S) *and* (P, T) *are Compatible,*
(iii) there exists a sequence $\{x_n\}$ *which is asymptotically regular with respect to* (P, S),(S, T) *and* (P, T),
(iv) S and T are Continuous,
Then P,S and T have a unique common fixed point.

Proof: Proof: The proof follows from Theorem 3.5.

4 Open Problem

Question 1. Are the above mentioned theorems true in an intuitionistic fuzzy 2-metric space?.

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