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Cartesian product and homomorphism of interval-valued fuzzy linear space

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Abstract

The aim of this paper is to introduce the notion of cartesian product and homomorphism of interval-valued fuzzy linear space and to provide some results on it.

Keywords: *fuzzy field, fuzzy linear space, interval-valued fuzzy field, interval-valued fuzzy linear space.*

1 Introduction

After Zadeh's[20] introduction of interval-valued fuzzy sets, where the values of the membership functions are interval of real numbers instead of the real points,there was much important in this field. G.Lubczonok and V.Muralli[9] introduced an interesting theory of flags and fuzzy subspaces of vector spaces. S.Vijayabalaji, S.Anitha Shanthi and N.Thillaigovindan[17] introduced the notions of interval-valued fuzzy subspace and interval-valued fuzzy n-normed linear spaces. Nanda[12] introduced the concepts of fuzzy fields and fuzzy linear spaces. Gu Wenxiang and Lu Tu[5] redefined the concept of fuzzy fields and fuzzy linear spaces. T.K.Samanta and I.H.Jebril[15] introduced the notion of finite dimensional intuitionistic fuzzy normed linear space. Recently, S.Vijayabalaji and S.Sivaramakrishnan[18] introduced the notion of interval-valued fuzzy field and interval-valued fuzzy linear space.

In this paper we introduce the notion of cartesian product and homomorphism of interval-valued fuzzy linear space and provide results on it.

2 Preliminaries

In the following we provide the essential definitions and results necessary for the development of our theory.

Definition 2.1[20]. An interval number on $[0,1]$, say $\overline{a}$, is a closed sub interval of $[0,1]$, that is $\overline{a}=[a^-,a^+]$, where $0\le a^-\le a^+\le 1$. Let $D[0,1]$ denote the family of all closed sub-intervals of $[0,1]$, that is,
$D[0,1]=\{\overline{a}=[a^-,a^+]:a^-\le a^+ \text{ and } a^-,a^+\in[0,1]\}$.

Definition 2.2[20]. Let $\overline{a_i}=[a_i^-,a_i^+]\in D[0,1]$ for all $i\in\Omega$, Ω be an index set. Define
(a)$\inf^i\{\overline{a_i}:i\in\Omega\}=[\inf_{i\in\Omega}a_i^-,\inf_{i\in\Omega}a_i^+]$
(b)$\sup^i\{\overline{a_i}:i\in\Omega\}=[\sup_{i\in\Omega}a_i^-,\sup_{i\in\Omega}a_i^+]$.
In particular, whenever $\overline{a}=[a^-,a^+]$, $\overline{b}=[b^-,b^+]$ in $D[0,1]$, we define
(i)$\overline{a}\le\overline{b}$ if and only if $a^-\le b^-$ and $a^+\le b^+$
(ii)$\overline{a}=\overline{b}$ if and only if $a^-=b^-$ and $a^+=b^+$
(iii)$\overline{a}<\overline{b}$ if and only if $a^-<b^-$ and $a^+<b^+$.
(iv)$\min^i\{\overline{a},\overline{b}\}=[\min\{a^-,b^-\},\min\{a^+,b^+\}]$
(v)$\max^i\{\overline{a},\overline{b}\}=[\max\{a^-,b^-\},\max\{a^+,b^+\}]$.

Definition 2.3[20]. Let X be a set. A mapping $\overline{A}:X\to D[0,1]$ is called an interval-valued fuzzy subset(briefly, an i-v fuzzy subset) of X, where
$\overline{A}(x)=[A^-(x),A^+(x)]$, for all $x\in X$, A^- and A^+ are fuzzy subsets in X such that $A^-(x)\le A^+(x)$ for all $x\in X$.

Definition 2.4[6]. Let V denote a vector space of dimension n over a field F. A fuzzy subspace is a fuzzy subset μ of V such that
$\mu(\alpha x+\beta y)\ge\min\{\mu(x),\mu(y)\}$,$x,y\in V$,$\alpha,\beta\in$F$(Field)$.

Definition 2.5[17]. Let V denote a vector space over a field F. Let
$\overline{A}:X\to D[0,1]$ be an interval-valued fuzzy subset of V. Then $\overline{A}$ is said to be an interval-valued fuzzy subspace (or shortly i-v fuzzy subspace) if,
$\overline{A}(\alpha x+\beta y)\ge\min^i\{\overline{A}(x),\overline{A}(y)\}$, $x,y\in V$ and $\alpha,\beta\in$F$(field)$.

Definition 2.6[4]. Let f be a mapping from a set X into a set Y. Let B be an interval-valued fuzzy set in Y. Then the inverse image of B,i.e., $f^{-1}[B]$ is the interval-valued fuzzy set in X with the membership function given by $\overline{\mu}_{f^{-1}[B]}(x) = \overline{\mu}_B(f(x))$, $\forall x \in X$.

Definition 2.7[4]. Let f be a mapping from a set X into a set Y. Let A be an interval-valued fuzzy set in X. Then the image of A, i.e., $f[A]$ is the interval-valued fuzzy set in Y with the membership function defined by

$$\overline{\mu}_{f[A]}(y) = \begin{cases} r\sup\limits_{z\in f^{-1}(y)} \overline{\mu}_A(z), & if\ \ f^{-1}(y) \neq \phi, \forall y \in Y, \\ \qquad\qquad [0,0], & otherwise, \end{cases}$$

where $f^{-1}(y) = \{x : f(x) = y\}$ and $[0,0]$ denotes the interval-valued fuzzy empty set.

Definition 2.8[11]. Let A=$(\tilde{\mu}_A, \tilde{\lambda}_A)$ and B=$(\tilde{\mu}_B, \tilde{\lambda}_B)$ be interval-valued intuitionistic fuzzy sets on L. Then generalized cartesian product $A \times B$ is defined as follow: $A \times B$=$(\tilde{\mu}_A \times \tilde{\mu}_B, \widetilde{\lambda_A \times \lambda_B})$, where $(\tilde{\mu}_A \times \tilde{\mu}_B)(x, y)$=min$\{\tilde{\mu}_A(x), \tilde{\mu}_B(y)\}$ and $(\tilde{\lambda}_A \times \tilde{\lambda}_B)(x, y)$=max$\{\tilde{\lambda}_A(x), \tilde{\lambda}_B(y)\}$.

Definition 2.9[18]. Let X be a field and $\overline{A}$ be an interval-valued fuzzy set on X. If the following conditions hold:

$(i)\overline{A}(x + y) \geq \min\{\overline{A}(x), \overline{A}(y)\}$,$x, y \in X$;
$(ii)\overline{A}(-x) = \overline{A}(x)$,$x \in X$;
$(iii)\overline{A}(xy) \geq \min\{\overline{A}(x), \overline{A}(y)\}$, $x, y \in X$;
$(iv)\overline{A}(x^{-1}) = \overline{A}(x)$, $(x \neq 0) \in X$.

Then $\overline{A}$ is said to be an interval-valued fuzzy field on X or briefly i-v fuzzy field on X, denoted by $(\overline{A}, X)$.

Example 2.10[18]. Consider a field $Z_5 = \{0, 1, 2, 3, 4\}$ with following Cayley tables:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

.	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Let $\overline{A} : Z_5 \to D[0,1]$ be an interval-valued fuzzy set defined by

$$\overline{A}(x) = \begin{cases} [0.8,\ 0.9], & if\ \ x = 0 \\ [0.6,\ 0.7], & otherwise \end{cases}$$

Clearly $\overline{A}$ is an interval-valued fuzzy field on Z_5.

Definition 2.11[18]. Let X be a field and ($\overline{A}$,X) be an interval-valued fuzzy field of X. Let Y be an linear space over X and $\overline{V}$ be an interval-valued fuzzy set of Y. Suppose the following conditions hold:

$(i)\overline{V}(x+y) \geq \min\{\overline{V}(x), \overline{V}(y)\}$,$x, y \in Y$;

$(ii)\overline{V}(-x) = \overline{V}(x)$, $x \in Y$;

$(iii)\overline{V}(\lambda x) \geq \min\{\overline{A}(\lambda), \overline{V}(x)\}$, $\lambda \in X, x \in Y$;

$(iv)\overline{A}(1) \geq \overline{V}(0)$.

Then ($\overline{V}$,Y) is called an interval-valued fuzzy linear space or briefly i-v fuzzy linear space over ($\overline{A}$,X).

Example 2.12[18]. Consider a linear space $Z_3 = \{0, 1, 2\}$ with the following Cayley tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

.	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Here Z_3 is a subset of Z_5 and
Let $\overline{A} : Z_5 \to D[0,1]$ and

Let $\overline{V} : Z_3 \to D[0,1]$ be a fuzzy subsets defined by

$$\overline{A}(x) = \begin{cases} [0.8,\ 0.9], & if\ \ x = 0 \\ [0.6,\ 0.7], & otherwise \end{cases}$$

and

$$\overline{V}(x) = \begin{cases} [0.5,\ 0.6], & if\ \ x = 0 \\ [0.3,\ 0.4], & otherwise \end{cases}$$

Clearly $\overline{V}$ is an interval-valued fuzzy linear space of Z_3 over Z_5.

Definition 2.13[18]. Let ($\overline{V}$,Y) and ($\overline{W}$,Y) be two interval-valued fuzzy linear space over an interval-valued fuzzy field ($\overline{A}$,X). If $\overline{W} \subset \overline{V}$,then ($\overline{W}$,Y) is said to be an interval-valued fuzzy linear subspace of ($\overline{V}$,Y).

Theorem 2.14[18]. If ($\overline{A}$,X) is an interval-valued fuzzy field of X, then
$(i)\overline{A}(0) \geq \overline{A}(x)$, $x \in$X;
$(ii)\overline{A}(1) \geq \overline{A}(x)$, $x(\neq 0) \in$X.

Remark 2.15[18]. If ($\overline{A}$,X) is an interval-valued fuzzy field of X,then
$\overline{A}(0) \geq \overline{A}(1)$.

Remark 2.16[18]. If ($\overline{V}$,Y) is an interval-valued fuzzy linear space over ($\overline{A}$,X),then
$(i)\overline{A}(0) \geq \overline{V}(0)$;
$(ii)\overline{V}(0) \geq \overline{V}(x)$,$x \in$Y;
$(iii)\overline{A}(1) \geq \overline{V}(x)$, $x \in$Y.

Theorem 2.17[18]. Let ($\overline{A}$,X) be an interval-valued fuzzy field of X,and Y a linear space over X. Assume $\overline{V}$ is an interval-valued fuzzy set of Y. Then ($\overline{V}$,Y) is an interval-valued fuzzy linear space over($\overline{A}$,X) iff
(i) $\overline{V}(\lambda x+\mu y) \geq \min\{\min[\overline{A}(\lambda), \overline{V}(x)], \min[\overline{A}(\mu), \overline{V}(y)]\}$,$\lambda, \mu \in$X and $x, y \in$Y.
(ii) $\overline{A}(1) \geq \overline{V}(x)$, $x \in$Y.

Theorem 2.18[18]. The intersection of a family of interval-valued fuzzy linear spaces is an interval-valued fuzzy linear space.

3 Main results

We now introduce the notion of cartesian product of two interval-valued fuzzy linear spaces in the following theorem.

Theorem 3.1. Let ($\overline{A}$,X) be an interval-valued fuzzy field of X. Let $(\overline{V}_1, Y_1)$,$(\overline{V}_2, Y_2)$ be interval-valued fuzzy linear spaces over ($\overline{A}$,X). Then $(\overline{V}_1 \times \overline{V}_2,\, Y_1 \times Y_2)$ is an interval-valued fuzzy linear space over ($\overline{A}$,X).

Proof. Let $\overline{V} = \overline{V}_1 \times \overline{V}_2$.
Let $u = (u_1, u_2), v = (v_1, v_2) \in Y_1 \times Y_2$, and $\lambda, \mu \in$X
$(i)\overline{V}(\lambda u + \mu v)$

$$= (\overline{V}_1 \times \overline{V}_2)(\lambda u_1 + \mu v_1, \lambda u_2 + \mu v_2)$$
$$= \min_{j=1,2} \overline{V}_j(\lambda u_j + \mu v_j)$$
$$\geq \min_{j=1,2}\{\min[\overline{V}_j(\lambda u_j), \overline{V}_j(\mu v_j)]\}$$
$$\geq \min_{j=1,2}\{\min[\overline{A}(\lambda), \overline{V}_j(u_j)], \min[\overline{A}(\mu), \overline{V}_j(v_j)]\}$$
$$=\min\{\min[\overline{A}(\lambda), \min_{j=1,2} \overline{V}_j(u_j)], \min[\overline{A}(\mu), \min_{j=1,2} \overline{V}_j(v_j)]\}$$
$$=\min\{\min[\overline{A}(\lambda), \overline{V}(u)], \min[\overline{A}(\mu), \overline{V}(v)]\}$$

(ii) $\overline{A}(1) \geq \overline{V}_j(u_j)$ for all j=1,2.
So,

$$\overline{A}(1) \geq \min_{j=1,2} \overline{V}_j(u_j) = \overline{V}(u) \text{ for all } u \in Y_1 \times Y_2.$$

Hence $(\overline{V}_1 \times \overline{V}_2, Y_1 \times Y_2)$ is an interval-valued fuzzy linear space over ($\overline{A}$,X). □

Theorem 3.2. Let X_1 and X_2 be fields and $f : X_1 \to X_2$ be a homomorphism. Suppose that $(\overline{A}_1, X_1)$ is an interval-valued fuzzy field of X_1 and $(\overline{A}_2, X_2)$ is an interval-valued fuzzy field ofX_2. Then
$(i)(f(\overline{A}_1), X_2)$ is an interval-valued fuzzy field of X_2.
$(ii)(f^{-1}(\overline{A}_2), X_1)$ is an interval-valued fuzzy field of X_1.

Proof. (i) Let $u, v \in X_2$.
(a) If either $f^{-1}(u) = \phi$ or $f^{-1}(v) = \phi$, then their $f(\overline{A}_1)(u) = 0$ or $f(\overline{A}_1)(v) = 0$
So, $f(\overline{A}_1)(u + v) \geq 0 = \min\{f(\overline{A}_1)(u), f(\overline{A}_1)(v)\}$
Suppose that neither $f^{-1}(u) = \phi$ nor $f^{-1}(v) = \phi$.
Then $f^{-1}(u + v) \neq \phi$.
Let $r \in f^{-1}(u)$ and $s \in f^{-1}(v)$.
Then $r + s \in f^{-1}(u + v)$, so $r + s \in \{w : w \in f^{-1}(u + v)\}$.
Therefore $\{r + s : r \in f^{-1}(u), s \in f^{-1}(v)\} \subseteq \{w : w \in f^{-1}(u + v)\}$ (3.1)
Now, $f(\overline{A}_1)(u + v)$

$= \sup_{w \in f^{-1}(u+v)} \overline{A}_1(w).$
$\geq \sup_{r \in f^{-1}(u), s \in f^{-1}(v)} \overline{A}_1(r+s)$, by the expression (3.1)
$\geq \sup_{r \in f^{-1}(u), s \in f^{-1}(v)} \min\{\overline{A}_1(r), \overline{A}_1(s)\}$
$= \min\{\sup_{r \in f^{-1}(u)} \overline{A}_1(r), \sup_{s \in f^{-1}(v)} \overline{A}_1(s)\}$
$= \min\{f(\overline{A}_1)(u), f(\overline{A}_1)(v)\}$
$\Rightarrow f(\overline{A}_1)(u+v) \geq \min\{f(\overline{A}_1)(u), f(\overline{A}_1)(v)\}.$

(b) $f(\overline{A}_1)(-u)$
$= \sup\{\overline{A}(r) : f(r) = -u\} = \sup\{\overline{A}(r) : f(-r) = u\}$
$= \sup\{\overline{A}(-r) : f(-r) = u\} = \sup\{\overline{A}(s) : f(s) = u\}$
$= f(\overline{A}_1)(u).$

(c) As in (a), $f(\overline{A}_1)(uv) \geq \min\{f(\overline{A}_1)(u), f(\overline{A}_1)(v)\}$
(d) As in (b), if $u \neq 0$,then $f(\overline{A}_1)(u^{-1}) = f(\overline{A}_1)(u)$
Hence $(f(\overline{A}_1), X_2)$ is an interval-valued fuzzy field of X_2.

(ii) Let $r, s \in X_1$.
$f^{-1}(\overline{A}_2)(r+s)$
$= \overline{A}_2(f(r+s))$
$= \overline{A}_2(f(r) + f(s))$ (since f is homomorphism)
$\geq \min\{\overline{A}_2(f(r)), \overline{A}_2(f(s))\}$
$= \min\{f^{-1}(\overline{A}_2)(r), f^{-1}(\overline{A}_2)(s)\}$
and
$f^{-1}(\overline{A}_2)(-r)$
$= \overline{A}_2(f(-r)) = \overline{A}_2(-f(r))$
$= \overline{A}_2(f(r))$
$= f^{-1}(\overline{A}_2)(r).$
Similarly,
$f^{-1}(\overline{A}_2)(rs) \geq \min\{f^{-1}(\overline{A}_2)(r), f^{-1}(\overline{A}_2)(s)\}$ and
$f^{-1}(\overline{A}_2)(r^{-1}) = f^{-1}(\overline{A}_2)(r)$ if $r \neq 0$.
Hence $(f^{-1}(\overline{A}_2), X_1)$ is an interval-valued fuzzy field of X_1. □

Theorem 3.3. Let Y and Z be linear spaces over the field X, and f a linear transformation of Y into Z. Let ($\overline{A}$,X) be an interval-valued fuzzy field of X, and ($\overline{W}$,Z) be an interval-valued fuzzy linear space over($\overline{A}$,X). Then ($f^{-1}(\overline{W})$,Y) is an interval-valued fuzzy linear space over ($\overline{A}$,X).

Proof. (i) For all $\lambda, \mu \in X$ and $u, v \in Y$,
$f^{-1}(\overline{W})(\lambda u + \mu v)$
$= \overline{W}(f(\lambda u + \mu v))$

$\geq \min\{\min[\overline{A}(\lambda), \overline{W}(f(u))], \min[\overline{A}(\mu), \overline{W}(f(v))]\}$
$=\min\{\min[\overline{A}(\lambda), f^{-1}(\overline{W})(u)], \min[\overline{A}(\mu), f^{-1}(\overline{W})(v)]\}$

(ii) Since ($\overline{W}$,Z) is an interval-valued fuzzy linear space over ($\overline{A}$,X), for all $u \in Y$, $\overline{A}(1) \geq \overline{W}(f(u)) = f^{-1}(\overline{W})(u)$
Therefore, ($f^{-1}(\overline{W})$,Y) is an interval-valued fuzzy linear space over ($\overline{A}$,X). □

Theorem 3.4. Let Y and Z be linear spaces over the field X, and f a linear transformation of Y into Z. Let ($\overline{A}$,X) be an interval-valued fuzzy field of X and ($\overline{V}$,Y) be an interval-valued fuzzy linear space over ($\overline{A}$,X). Then ($f(\overline{V})$,Z) is an interval-valued fuzzy linear space over ($\overline{A}$,X).

Proof. For all $\lambda, \mu \in X$ and $u, v \in Z$, if either $f^{-1}(u)$ or $f^{-1}(v)$ is empty,

then either $f(\overline{V})(u) = 0$ or $f(\overline{V})(v) = 0$,
so $f(\overline{V})(\lambda u + \mu v) \geq 0 = \min\{\min[\overline{A}(\lambda), f(\overline{V})(u)], \min[\overline{A}(\mu), f(\overline{V})(v)]\}$,
$\lambda, \mu \in$X and $x, y \in$Y, the inequality (i) of Theorem 2.17 is satisfied.

Suppose neither $f^{-1}(u)$ nor $f^{-1}(v)$ is empty, then $f^{-1}(\lambda u + \mu v) \neq \phi$.
Let $r \in f^{-1}(u), s \in f^{-1}(v)$.
Then
$f(\lambda r + \mu s) = \lambda f(r) + \mu f(s) = \lambda u + \mu v$.
So,
$f(\overline{V})(\lambda u + \mu v)$

$$
\begin{aligned}
&= \sup_{w \in f^{-1}(\lambda u + \mu v)} \overline{V}(w) \\
&\geq \sup_{r \in f^{-1}(u), s \in f^{-1}(v)} \overline{V}(\lambda r + \mu s) \\
&\geq \sup_{r \in f^{-1}(u), s \in f^{-1}(v)} \min\{\min[\overline{A}(\lambda), \overline{V}(r)], \min[\overline{A}(\mu), \overline{V}(s)]\} \\
&= \min\{\min[\overline{A}(\lambda), \sup_{r \in f^{-1}(u)} \overline{V}(r)], \min[\overline{A}(\lambda), \sup_{s \in f^{-1}(v)} \overline{V}(s)]\} \\
&= \min\{\min[\overline{A}(\lambda), f(\overline{V})(u)], \min[\overline{A}(\mu), f(\overline{V})(v)]\}
\end{aligned}
$$

Obviously, for any $u \in$Z, $\overline{A}(1) \geq f(\overline{V})(u)$.
Thus ($f(\overline{V})$,Z) is an interval-valued fuzzy linear space over ($\overline{A}$,X),which ends the proof. □

Theorem 3.5. If $(\overline{V}, Y)$ is an interval-valued fuzzy linear space over ($\overline{A}$,X), then the nonempty set $X_V = \min\{a \in$X: $\overline{A}(a) \geq \overline{V}(u), \forall u \in$Y$\}$ is a subfield of X. Also, $(\overline{A}, X_V)$ is an interval-valued fuzzy field of X_V and ($\overline{V}$,Y) is an interval-valued fuzzy linear space over $(\overline{A}, X_V)$.

Proof. Let $a, b \in X_V$.
Then for all $u \in Y$,
$\overline{A}(a-b) \geq \min\{\overline{A}(a), \overline{A}(b)\} \geq \overline{V}(u)$ and for $a, b \neq 0$,
$\overline{A}(ab^{-1}) \geq \min\{\overline{A}(a), \overline{A}(b)\} \geq \overline{V}(u)$
Therefore $a - b \in X_V$ and $ab^{-1} \in X_V$ if $a, b \neq 0$.
Therefore X_V is a subfield of X.

The second part is trivial. □

4 Open Problem

We suggest some Open Problems as follows:

(i) Construction of interval-valued fuzzy norm on interval-valued fuzzy linear space.

(ii) Construction of interval-valued fuzzy inner product on interval-valued fuzzy linear space.

(iii) Interrelating interval-valued fuzzy norm and interval-valued fuzzy inner product space.

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