ON A CERTAIN SUBCLASS OF ANALYTIC FUNCTIONS WITH DIFFERENTIAL EQUATION AND SUBORDINATION

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Received 1 April 2015; Accepted 2 July 2015

Abstract

The main purpose of this paper is to further investigate the subclass $T_{m}^{l}(\alpha, \beta, \gamma)$ by applying the differential subordination theorem. Also obtained are subordination results on a convex function and an incomplete beta function. Furthermore, we discussed certain subordination results for the function that belongs to the class $T_{m}^{l}(\alpha, \beta, \gamma)$ with a Cauchy-Euler differential equation.

Keywords: Subordination, Analytic functions, Convex function, Differential equation, Subordinating factor sequence, Hadamard product (or Convolution).

2000 Mathematical Subject Classification: 30C45.
1 Introduction

Let $A$ be the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are analytic in the unit disk $U = \{ z : |z| < 1 \}$ and normalized by $f(0) = f'(0) - 1 = 0$.

Also let $S$ be the subclass of $A$ consisting of univalent functions and $K$ denotes the class of convex functions such that

$$K = \left\{ f \in A : \text{Re} \frac{zf''(z)}{f'(z)} + 1 > 0, z \in U \right\}.$$ 

Furthermore, we denote $T$ as the subclass of $S$ whose elements can be expressed in the form

$$f(z) = z - \sum_{n=2}^{\infty} |a_n| z^n$$

introduced and studied by Silverman [1].

In 2010, Murugusundaramorthy and Magesh [2] introduced and studied the class of functions $f(z)$ defined by (2) satisfying the following conditions:

$$\text{Re} \left\{ 1 + \frac{1}{\gamma} \left( \frac{z(H_m^l[\alpha_1]f(z))'}{H_m^l[\alpha_1]f(z)} - \alpha \right) \right\} > \beta \left| 1 + \frac{1}{\gamma} \left( \frac{H_m^l(\alpha_1)f(z)}{H_m^l[\alpha_1]f(z)} - 1 \right) \right|, z \in U$$

for $-1 \leq \alpha < 1$, $\beta \geq 0$, and $\gamma$ is a non-zero complex.

This class is denoted by $TS_m^l(\alpha, \beta, \gamma)$.

We note that

$$H_m^l[\alpha_1]f(z) := z + \sum_{n=2}^{\infty} \Gamma_n a_n z^n,$$

and

$$\Gamma_n = \frac{(\alpha_1)_{n-1} \cdots (\alpha_l)_{n-1}}{(\beta_1)_{n-1} \cdots (\beta_m)_{n-1}} \frac{1}{(n-1)!}.$$  

For this class, the authors obtained some geometric properties such as coefficient estimates, extreme points, the radii of close-to-convexity, starlikeness, convexity and neighbourhood results (for details see, [2]).

In this paper, motivated by the techniques in [3], we further investigate the class $TS_m^l(\alpha, \beta, \gamma)$ by applying the subordination differential theorem.
2 Preliminaries and Definitions

We state some basic results which are relevant to our main results and also give certain fundamental definitions.

**Definition 1:** Let \( f(z) \) and \( g(z) \) be analytic in \( U \), where \( f(z) \) is as given in (1) and \( g(z) \) is defined as

\[
g(z) = z + \sum_{n=2}^{\infty} b_n z^n
\]

(6)

The function \( f(z) \) is subordinate to \( g(z) \) (written as \( f \prec g \)) if there exists a function \( \varphi(z) \) analytic (not necessarily univalent) in \( U \) and satisfying \( \varphi(0) = 0, |\varphi(z)| < 1 \) such that \( f(z) = g(\varphi(z)) \) for \( z \in U \).

**Definition 2:** (Hadamard product or convolution)

Given two functions \( f(z) \) and \( g(z) \) where \( f(z) \) is as defined in (1) and \( g(z) \) is given by (6), then the Hadamard product (or convolution) \( f \ast g \) of \( f(z) \) and \( g(z) \) is defined by

\[
(f \ast g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n
\]

(7)

The function \( (f \ast g)(z) \) is also analytic in \( U \).

**Definition 3:** (Subordinating factor sequence)

A sequence \( \{c_n\}_{n=1}^{\infty} \) of complex numbers is said to be a subordinating factor sequence if whenever \( f(z) \) of the form (1) is analytic, univalent and convex in \( U \), the subordination is given by

\[
\sum_{n=1}^{\infty} a_n c_n z^n \prec f(z), z \in U, a_1 = 1.
\]

(8)

**Theorem 1:** [4]
The sequence \( \{b_n\}_{n=1}^{\infty} \) is a subordinating factor sequence if and only if

\[
Re\left\{1 + 2 \sum_{n=1}^{\infty} c_n z^n\right\} > 0, z \in U.
\]

(9)

**Theorem 2:** [5]
Let \( 0 < a \leq c \). If \( c \geq 2 \) or \( a + c \geq 3 \), then the function

\[
h(a, c; z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} z^n, (z \in U)
\]

(10)

belongs to the class \( K \) of convex functions.
In [2], Murugusundaramorthy and Magesh proved the necessary and sufficient condition for functions \( f \in T \) to be in the class \( TS^l_m(\alpha, \beta, \gamma) \)

which is equivalent to the following Theorem:

**Theorem 3:** [2]

A function \( f \in T \) is in the class \( TS^l_m(\alpha, \beta, \gamma) \) if and only if

\[
\sum_{n=2}^{\infty} \left[ (n + |\gamma|)(1 - \beta) - (\alpha - \beta) \right] |a_n| \leq (1 - \alpha) + |\gamma|(1 - \beta),
\]

where \(-1 \leq \alpha < 1, \beta \geq 0, \gamma \in C\{0\}\) and \( \Gamma_n \) is as defined in (5).

**Theorem 4:** [6] If the function \( f(z) \) and \( g(z) \) are analytic in \( U \) with \( g(z) \prec f(z) \), then for \( p > 0 \) and \( z = re^{i\theta}, (0 < r < 1) \), we have

\[
\int_0^{2\pi} |f(z)|^p d\theta \leq \int_0^{2\pi} |g(z)|^p d\theta
\]

**3 Main Results**

**3.1 CERTAIN SUBORDINATION PROPERTIES OF THE CLASS \( TS^l_m(\alpha, \beta, \gamma) \)**

We begin with the following theorem: **Theorem 5:**

Let \( f \in TS^l_m(\alpha, \beta, \gamma) \), then

\[
\frac{A}{2(A + B)} (f * g)(z) \prec g(z)
\]

and

\[
\frac{A}{2(A + B)} \int_0^{2\pi} |(f * g)(re^{i\theta})|^p d\theta
\]

\[
\leq 2 \int_0^{2\pi} |g(re^{i\theta})|^p d\theta.
\]

where

\[
A = \left[ (2 + |\gamma|)(1 - \beta) - (\alpha - \beta) \right] \Gamma_2
\]

and

\[
B = \left[ (1 - \alpha) + |\gamma|(1 - \beta) \right];
\]
for $p > 0$, $0 < |z| = r < 1$, and $g \in K$.

The constant factor
\[
\frac{A}{2(A + B)} = \frac{[(2 + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_2}{2[(2 + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_2 + [(1 - \alpha) + |\gamma|(1 - \beta)]}
\]

in the subordination result (13) cannot be replaced by a larger one.

Moreover the result is sharp for the function
\[
f_0(z) = z - \frac{1}{A}(Az - Bz^2). \tag{15}
\]

**Proof** Let $f(z) \in TS^l_n(\alpha, \beta, \gamma)$, then
\[
\frac{A}{2(A + B)}(f \ast g)(z) = \frac{A}{2(A + B)}\left(z + \sum_{n=2}^{\infty} a_n b_n z^n\right) \tag{16}
\]

where $g(z) = \sum_{n=2}^{\infty} b_n z^n \in K$.

By Theorem 1, it is sufficient to show that
\[
Re\left\{1 + 2 \sum_{n=1}^{\infty} \frac{A}{2(A + B)} a_n z^n\right\} = Re\left\{1 + \sum_{n=1}^{\infty} \frac{A}{(A + B)} a_n z^n\right\} > 0, \quad z \in U \tag{17}
\]

in order that the subordination (13) should hold true.

Thus it implies that the sequence
\[
\left\{\frac{A}{(A + B)} a_n\right\}_{n=1}^{\infty}
\]

is a subordinating factor sequence, with $a_1 = 1$.

Now,
\[
Re\left\{1 + \sum_{n=1}^{\infty} \left(\frac{A}{(A + B)}\right) a_n z^n\right\} = Re\left\{1 + \left(\frac{A}{A + B}\right) a_1 z + \frac{1}{A + B} \sum_{n=2}^{\infty} A a_n z^n\right\} \geq 1 - \frac{A}{A + B} r - \frac{1}{A + B} \sum_{n=2}^{\infty} A |a_n| r^n \tag{18}
\]
On a Certain Subclass of Analytic Functions

Since $\psi(n) = [(n + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_n, (n = 2, 3, \ldots)$ is an increasing function of $n$, so $0 < A \leq \psi(n), (n = 2, 3, \ldots)$ where $\psi(n) = [(n + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_n$; we have

$$
Re\left\{1 + \sum_{n=1}^{\infty} \left(\frac{A}{A + B}\right) a_n z^n\right\} = Re\left\{1 + \left(\frac{A}{A + B}\right) a_1 z + \frac{1}{A + B} \sum_{n=2}^{\infty} \psi(n) a_n z^n\right\} > 1 - \left(\frac{A}{A + B}\right) r - \left(\frac{B}{A + B}\right) r = 1 - r > 0
$$

(19)

which establishes (17) and consequently establish (13).

Note that by (11)

$$
\sum_{n=2}^{\infty} \psi(n) |a_n| = \sum_{n=2}^{\infty} [(n + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_n |a_n| \leq (1 - \alpha) + |\gamma|(1 - \beta) = B
$$

The inequality (14) follows from (13) and Theorem 4.

In order to prove the sharpness of the constant factor

$$
\frac{A}{2(A + B)} = \frac{2[(2 + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_2}{[(2 + |\gamma|)(1 - \beta) - (\alpha - \beta)\Gamma_2 + [(1 - \alpha) + |\gamma|(1 - \beta)]},
$$

we consider the function $f_0(z) \in TS^l_{m}(\alpha, \beta, \gamma)$ given by (15). Thus from (13), we have

$$
\frac{A}{2(A + B)} f_0(z) \prec \frac{z}{1 - z}, (z \in U)
$$

(20)

by taking $g(z) = \frac{z}{1 - z} = z + \sum_{n=2}^{\infty} z^n$.

Moreover, it can easily be verified for the function $f_0(z)$ given by (15) that

$$
\min\left\{Re\left\{\frac{A}{2(A + B)} f_0(z)\right\}\right\} = -\frac{1}{2}, (z \in U).
$$

(21)

This shows that the constant

$$
\frac{A}{A + B} = \frac{[(2 + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_2}{[(2 + |\gamma|)(1 - \beta) - (\alpha - \beta)\Gamma_2 + [(1 - \alpha) + |\gamma|(1 - \beta)]},
$$
is the best possible. This completes the proof of Theorem 5.

**Corollary 6**

Let \( f(z) \in TS_m^l(\alpha, \beta, \gamma) \) and

\[
F(z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} a_n z^n
\]

where \((f * h(a, c; z))(z) = F(z)\) then

\[
\frac{A}{A+B} (f * h(a, c; z))(z) < 2h(a, c; z).
\]  \hspace{1cm} (22)

and

\[
Ref(z) > -\left( \frac{A+B}{A} \right)
\]  \hspace{1cm} (23)

where \(h(a, c; z)\) is as defined in (10)

**Proof.**

Using Theorem 5, we have that

\[
h(a, c; z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} a_n z^n \in K,
\]

Since \(0 < a \leq c, c \geq 2\) or \(a + c \geq 3\).

Now, by taking \(g(z) = h(a, c; z) = z + \sum_{n=2}^{\infty} z^n\) in theorem 1, respectively, the result (22) and (23) are obtained.

**Corollary 6**

Let \( f(z) \in TS_m^l(\alpha, \beta, \gamma) \) in \( U \) and \( p > 0, 0 < |z| = r < 1 \), then for function \( g \in K \)

\[
\frac{4\Gamma_n}{3 + 4\Gamma_n} (f * g)(z) < 2g(z)
\]  \hspace{1cm} (24)

and

\[
\frac{4\Gamma_n}{3 + 4\Gamma_n} \int_{0}^{2\pi} |(f * g)(re^{i\theta})|^p d\theta \leq 2 \int_{0}^{2\pi} |g(re^{i\theta})|^p d\theta.
\]  \hspace{1cm} (25)

**Proof.**

By taking \(\gamma = -1, \alpha = -1\) and \(\beta = 0\) in Theorem 5, Corollary 6 is obtained.

**Remark 3.1.** For \(\gamma = \pm i, \alpha = -1\) and \(\beta = 0\) in Theorem 5 we obtain the result in Corollary 6.

**Corollary 7**

Let \( f(z) \in TS_m^l(0, 0, \pm i) \) in \( U \) and \( p > 0, 0 < |z| = r < 1 \), then for function \( g \in K \)

\[
\frac{3\Gamma_n}{2 + 3\Gamma_n} (f * g)(z) < 2g(z)
\]  \hspace{1cm} (26)
and
\[ \frac{3\Gamma_n}{2 + 3\Gamma_n} \int_0^{2\pi} |(f \ast g)(re^{i\theta})|^p d\theta \leq 2 \int_0^{2\pi} |g(re^{i\theta})|^p d\theta. \] (27)

### 3.2 OTHER SUBORDINATION PROPERTIES OF THE CLASS \( TS^l_m(\alpha, \beta, \gamma) \) WITH FIXED EQUATION

Our next results in this section are on the functions in the class \( TS^l_m(\alpha, \beta, \gamma) \) which are associated with the following non-homogeneous Cauchy-Euler differential
\[
z^2 \frac{d^2q}{dz^2} + 2(\mu + 1)z \frac{dq}{dz} + \mu(\mu + 1)q
= (\mu + 1)(\mu + 2)z + (\mu + 2)(\mu + 3) \sum_{n=2}^{\infty} c_n z^n
\] (28)
for \( q(z) \in T, f(z) \in TS^l_m(\alpha, \beta, \gamma), \mu + 1 > 0, \mu \in \mathbb{R}. \)

**Remark 3.2.** Equation (28) corrects the one in [3]

O. Altintas, et al. [7] had earlier used Cauchy-Euler differential equation to study the distortion inequalities and neighbourhood problems of the other class of functions.

**Theorem 8:** Let the function \( q(z) = z + \sum_{n=2}^{\infty} c_n z^n \) be in \( T \) and satisfy the equation (28) with \( f(z) \in TS^l_m(\alpha, \beta, \gamma), \) then
\[
\frac{A(\mu + 3)}{B(\mu + 1) + (\mu + 3)A} (q \ast g)(z) < 2g(z)
\] (29)
and
\[
\frac{A(\mu + 3)}{B(\mu + 1) + (\mu + 3)A} \int_0^{2\pi} |(q \ast g)(re^{i\theta})|^p d\theta \leq 2 \int_0^{2\pi} |g(re^{i\theta})|^p d\theta\] (30)
for function \( g \in K, 0 < |z| = r < 1, p > 0. \) and \( \psi(2) \) as defined earlier.

**Proof** Suppose \( g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in K, \) then
\[
\frac{A(\mu + 3)}{B(\mu + 1) + (\mu + 3)A} (q \ast g)(z) = \frac{A(\mu + 3)}{2[A(\mu + 3) + B(\mu + 1)]} \left( z + \sum_{n=2}^{\infty} b_n c_n z^n \right)
\]
\[
= \frac{A(\mu + 3)}{2[A(\mu + 3) + B(\mu + 1)]} z + \frac{A(\mu + 3)}{2[A(\mu + 3) + B(\mu + 1)]} \sum_{n=2}^{\infty} b_n c_n z^n.
\] (31)
By Theorem 1, it is sufficient to show that

\[ Re \left\{ 1 + 2 \sum_{n=1}^{\infty} \frac{A(\mu + 3)}{2[A(\mu + 3) + B(\mu + 1)]} c_n z^n \right\} \]

\[ = Re \left\{ 1 + \sum_{n=1}^{\infty} \frac{A(\mu + 3)}{A(\mu + 3) + B(\mu + 1)} c_n z^n \right\} > 0, \]  

(32)

\[ z \in U \] inorder that the subordination (29) should hold true.

Thus it implies that the sequence

\[ \left\{ \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]} c_n \right\}_{n=1}^{\infty} \]

is a subordinating factor sequence, with \( a_1 = 1 \).

Now,

\[ Re \left\{ 1 + \sum_{n=1}^{\infty} \left( \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]} \right) a_n z^n \right\} \]

\[ = Re \left\{ 1 + \left( \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]} \right) a_1 z \right. \]

\[ + \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]} \sum_{n=2}^{\infty} \psi(2) c_n z^n \right\} \]

\[ \geq 1 - \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]} r - \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]} \sum_{n=2}^{\infty} |c_n| r^n \]

(33)

Because \( L(z) \) satisfies the differential equation with the \( f(z) \in TS_m^l(\alpha, \beta, \gamma) \), so

\[ c_n = \frac{(\mu + 1)(\mu + 2)}{(n + \mu)(n + \mu + 1)} a_n. \]
Following (33), we have

\[
\geq 1 - \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]^r} - \frac{(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]} \sum_{n=2}^{\infty} A \frac{(\mu + 1)(\mu + 2)}{(n + \mu)(n + \mu + 1)} |a_n|r^n
\]

\[
\geq 1 - \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]^r} - \frac{(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]} \sum_{n=2}^{\infty} A \frac{(\mu + 1)(\mu + 2)}{(2 + \mu)(\mu + 3)} |a_n|r^n
\]

\[
1 - \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]^r} - \frac{(\mu + 1)}{[A(\mu + 3) + B(\mu + 1)]} \sum_{n=2}^{\infty} A |a_n|r^n
\]

Since \(\psi(n) = [(n + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_n, (n = 2, 3, ...)\) is an increasing function of \(n\), so \(0 < A \leq \psi(n), (n = 2, 3, ...)\) where \(\psi(n) = [(n + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_n\). Following (34) we have

\[
> Re \left\{ 1 - \frac{A(\mu + 3)}{[A(\mu + 3) + B(\mu + 1)]^r} - \frac{(\mu + 1)}{[A(\mu + 3) + B(\mu + 1)]} \sum_{n=2}^{\infty} \psi(n) |a_n|r^n \right\}
\]

\[
> 1 - \frac{A(\mu + 3) + (\mu + 1)[(1 - \alpha) + |\gamma|(1 - \beta)]}{(\mu + 1)[(1 - \alpha) + |\gamma|(1 - \beta)] + A(\mu + 3)} r
\]

\[
= 1 - r > 0 \text{ since } 0 < r < 1.
\]

which thus establishes (32) and consequently establish (29).

Note that by (11)

\[
\sum_{n=2}^{\infty} \psi(n) |a_n| = \sum_{n=2}^{\infty} [(n + |\gamma|)(1 - \beta) - (\alpha - \beta)]\Gamma_n |a_n| \leq (1 - \alpha) + |\gamma|(1 - \beta) = B
\]

The inequality (30) follows from (13) and Theorem 4.

**Corollary 9**

Let \(q(z) = \sum_{n=2}^{\infty} c_n z^n \in T\) satisfy (28) with \(f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in TS_m^l(\alpha, \beta, \gamma)\) and \(F(z) = z + \sum_{n=2}^{\infty} \frac{a_{n-1}}{(c)_{n-1}} a_n z^n\), then
\[ A(\mu + 3) + (\mu + 1)[(1 - \alpha) + |\gamma|(1 - \beta)] F(z) \prec 2h(a, c; z) \] (36) and
\[ Req(z) > -\left( \frac{A(\mu + 3) + (\mu + 1)[(1 - \alpha) + |\gamma|(1 - \beta)]}{A(\mu + 3)} \right). \] (37)

where \( h(a, c; z) \) is as defined in (10) with \( 0 < a \leq c, c \geq 2 \) or \( a + c \geq 3 \) and \( |z| = r < 1, p > 0 \).

**Proof**
Since \( 0 < a \leq c, c \geq 2 \) or \( a + c \geq 3 \), using Theorem 5, we have that
\[ h(a, c; z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} z^n \in K. \]

Taking \( g(z) = h(a, c; z) = z + \sum_{n=2}^{\infty} z^n \) in theorem 8, respectively, the results (36) and (37) are obtained.

By taking \( \beta = 0, \gamma = \pm i \), in Theorem 8 we have the following:

**Corollary 10**
Let the function \( q(z) = z + \sum_{n=2}^{\infty} c_n z^n \) satisfy (28) with \( f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in TS_m^l(\alpha, 0, \pm i) \), then for \( g \in K \)
\[ \frac{(\mu + 3)(3 - \alpha)\Gamma_2}{(\mu + 1)(2 - \alpha) + (\mu + 3)(3 - \alpha)\Gamma_2} (q \ast g)(z) < 2g(z), z \in U \] (38)
and
\[ \frac{(\mu + 3)(3 - \alpha)\Gamma_2}{(\mu + 1)(2 - \alpha) + (\mu + 3)(3 - \alpha)\Gamma_2} \int_0^{2\pi} |(q \ast g)(re^{i\theta})|^p d\theta \leq 2 \int_0^{2\pi} |g(re^{i\theta})|^p d\theta. \] (39)

By taking \( \alpha = -1, \gamma = \pm i \), in Theorem 8 we have the following:

**Corollary 11**
Let \( q(z) \) satisfy (28) with \( f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in TS_m^l(\alpha, 0, \pm i) \), then for \( g \in K \)
\[ \frac{2(\mu + 3)(2 - \beta)\Gamma_2}{(\mu + 1)(2 - \beta) + 2(\mu + 3)(2 - \beta)\Gamma_2} (q \ast g)(z) < 2g(z), z \in U \] (40)
and
\[ \frac{2(\mu + 3)(2 - \beta)\Gamma_2}{(\mu + 1)(2 - \beta) + 2(\mu + 3)(2 - \beta)\Gamma_2} \int_0^{2\pi} |(q \ast g)(re^{i\theta})|^p d\theta \leq 2 \int_0^{2\pi} |g(re^{i\theta})|^p d\theta. \] (41)
4 Open Problem

Some of the results obtained in this paper (e.g. Theorem 8 and the Corollaries arisen from it) may be extended further.

References


