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A Certain Class of Meromorphic Univalent Functions with Positive and Fixed Second Coefficients

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Abstract

In this paper we consider the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$ consisting of analytic meromorphic functions and with fixed second coefficients .In the present paper we have obtained coefficient inequalities for the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$. Also we have shown that this class is closed under arithmetic mean and convex linear combinations. Lastly we have obtained extreme points, growth and distortion bounds and the radius of meromorphically starlikeness for functions in the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$.

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1. Introduction

Let Σ denote the class of functions f of the form :

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \tag{1.1}$$

which are analytic and univalent in the punctured unit disc

$$U^* = \{z : z \in \mathbb{C}; |z| < 1\} = U \backslash \{0\}. \tag{1.2}$$

Let $\Sigma^*(\beta)$ and $\Sigma_c(\beta)$ be the subclasses of Σ consisting of univalent meromorphically starlike of order $\beta(0 \leq \beta < 1)$ and meromorphically convex of $\beta(0 \leq \beta < 1)$, respectively.
Analytically a function f of the form (1.1) is in the class $\Sigma^*(\beta)$ if and only if

$$-Re\left\{\frac{zf'(z)}{f(z)}\right\} > \beta \qquad (z \in U), \tag{1.3}$$

similarly, $f \in \Sigma_c(\beta)$ if and only if, f is of the form (1.1) and satisfies

$$-Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \beta \qquad (z \in U). \tag{1.4}$$

Note that

$$f \in \Sigma_c(\beta) \iff -zf' \in \Sigma^*(\beta).$$

The classes defined by Pomerenke [10], Clunie [5] and studied by Aouf and Silverman [3], Kaczmarski [7], Juneja and Reddy [6], Mogra [9] and others.

We define the class Σ_p of functions of the form:

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \quad (a_n \geq 0), \tag{1.5}$$

that are analytic and univalent in U^*.

Reddy et al. [11] introduced and defined the class $\Sigma(\lambda, \alpha, \beta, k)$ of meromorphic functions as follows:

Definition 1 [11]. A function $f \in \Sigma_p$ is said to be in the class $\Sigma(\lambda, \alpha, \beta, k)$ if

$$-Re\left\{\frac{zF'(z)}{F(z)} + \beta\right\} \geq k\left|\frac{zF'(z)}{F(z)} + 1\right| (z \in U), \tag{1.6}$$

where $0 \leq \alpha \leq \lambda < \frac{1}{2}, 0 \leq \beta < 1, k \geq 0$ and

$$F(z) = (1 - \lambda + \alpha)f(z) + (\lambda - \alpha)zf'(z) + \lambda\alpha z^2 f''(z).$$

It is noted that

$$\Sigma_p(\lambda, \alpha, \beta, k) = \Sigma(\lambda, \alpha, \beta, k) \cap \Sigma_p.$$

Remark 1. (i) Putting $\lambda = \alpha = 0$, we have $\Sigma_p(0, 0, \beta, k) = \Sigma^*(\beta, k)$ and $\lambda = 1, \alpha = 0$, we have $\Sigma_p(1, 0, \beta, k) = \Sigma^*(\beta, k)$, the classes of β- uniformly meromorphic functions were studied by Atshan and Kulkarni [4];
(ii) Putting $\lambda = \alpha = k = 0$, we have $\Sigma_p(0, 0, \beta, 0) = \Sigma^*(\beta)$ (see Aouf and Darwish [1], with $a_0 = 1$).

Motivated by Aouf and Darwish [1], Aouf and Joshi [2], Magesh et al. [8] and others, we shall study the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$.

We begin by recalling the following lemma due to Reddy et al. [10].

Lemma 1 [11]. The necessary and sufficient condition for a function $f \in \Sigma_p$ to be in the class $\Sigma_p(\lambda, \alpha, \beta, k)$ is given by

$$\sum_{n=1}^{\infty}[(n+\beta)+k\,(n+1)][(n-1)\,(n\lambda\alpha+\lambda-\alpha)+1]a_n \leq (1-\beta)\,(2\lambda\alpha-2\lambda+2\alpha+1)\,, \tag{1.7}$$

where $0 \leq \alpha \leq \lambda < \frac{1}{2}$, $0 \leq \beta < 1$, $k \geq 0$.
In view of (1.7), we can see that the function $f(z)$ defined by (1.5) in the class $\Sigma_p(\lambda, \alpha, \beta, k)$ satisfy the coefficient inequality

$$a_1 \leq \frac{(1-\beta)\,(2\lambda\alpha-2\lambda+2\alpha+1)}{2k+\beta+1}. \tag{1.8}$$

Hence we may take

$$a_1 = \frac{(1-\beta)\,(2\lambda\alpha-2\lambda+2\alpha+1)\,c}{2k+\beta+1}, 0 < c < 1. \tag{1.9}$$

Making use of (1.9), we now introduce the following class of functions:
Let $\Sigma_p(\lambda, \alpha, \beta, k, c)$ denote the subclass of $\Sigma_p(\lambda, \alpha, \beta, k)$ consisting of functions of the form:

$$f(z) = \frac{1}{z} + \frac{(1-\beta)\,(2\lambda\alpha-2\lambda+2\alpha+1)\,c}{2k+\beta+1}z + \sum_{n=2}^{\infty} a_n z^n \quad (0 < c < 1). \tag{1.10}$$

We note that:
$\Sigma_p(0, 0, \beta, 0, c) = \Sigma S^*_{0,c}(\beta)$ [1, with $a_0 = 1$].

In this paper we obtained coefficient inequalities for the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$, we have obtained extreme points, growth and distortion bounds. Also we have shown that this class is closed under arithmetic mean and convex linear combinations. Further, the radii of meromorphic starlikeness is obtained for the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$.

Body Math

Body Math **2. Main Results**

Theorem 1. Let the function $f(z)$ defined by (1.10). Then $f \in \Sigma_p(\lambda, \alpha, \beta, k, c)$, if and only if

$$\sum_{n=2}^{\infty}[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]a_n$$
$$\leq (1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c). \tag{2.1}$$

Proof. Putting

$$a_1 = \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}, 0 < c < 1, \tag{2.2}$$

In (1.7) and simplifying we get the result.
Remark 1. Putting $\lambda = \alpha = k = 0$ in Theorem 1, we obtain the result obtained by Aouf and Darwish [1, Theorem 1, with $a_0 = 1$].

Corollary 1. If the function f defined by (1.10) is in the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$, then

$$a_n \leq \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}, \qquad n \geq 2. \tag{2.3}$$

The estimates are sharp for the function $f(z)$ be given by

$$f(z) = \frac{1}{z} + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}z + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}z^n, \quad n \geq 2. \tag{2.4}$$

Next we prove the following growth and distortion properties for the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$.

Theorem 2. If the function defined by (1.10) is in the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$, then for $0 < |z| = r < 1$, we have

$$\frac{1}{r} - \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}r - \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}r^2 \leq |f(z)|$$
$$\leq \frac{1}{r} + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}r + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}r^2. \tag{2.5}$$

The result is sharp for the function $f(z)$ given by

$$f(z) = \frac{1}{z} + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}z + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}z^2. \tag{2.6}$$

Proof. Since $\Sigma_p(\lambda, \alpha, \beta, k, c)$, Theorem 1 yields

$$a_n \leq \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)(1-c)}{[(n+\beta) + k(n+1)][(n-1)(n\lambda\alpha + \lambda - \alpha) + 1]}, \qquad n \geq 2.$$

Thus for $0 < |z| = r < 1$, we have

$$\begin{aligned}
|f(z)| &\leq \frac{1}{|z|} + \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)c}{2k+\beta+1}|z| + \sum_{n=2}^{\infty} a_n |z|^n \\
&\leq \frac{1}{r} + \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)c}{2k+\beta+1}r + r^2\sum_{n=2}^{\infty} a_n \\
&\leq \frac{1}{r} + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}r + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}r^2,
\end{aligned}$$

and

$$\begin{aligned}
|f(z)| &\geq \frac{1}{|z|} - \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}|z| - \sum_{n=2}^{\infty} a_n |z|^n \\
&\geq \frac{1}{r} - \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)c}{2k+\beta+1}r - r^2\sum_{n=2}^{\infty} a_n \\
&\geq \frac{1}{r} - \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}r - \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}r^2.
\end{aligned}$$

Thus the proof of Theorem 2 is completed.

Theorem 3. If the function $f(z)$ given by (1.10) is in the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$, then for $0 < |z| = r < 1$, we have

$$\begin{aligned}
&\frac{1}{r^2} - \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1} - \frac{2(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}r \leq |f'(z)| \\
&\leq \frac{1}{r^2} + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1} + \frac{2(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}r. \qquad (2.7)
\end{aligned}$$

The result is sharp for the function $f(z)$ is given by (2.6).

Proof. In view of Theorem 1, it follows that

$$na_n \leq \frac{n(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}, \qquad n \geq 2. \qquad (2.8)$$

Thus, $0 < |z| = r < 1$ and making use of (2.8) we have

$$|f'(z)| \leq \left|\frac{-1}{z^2}\right| + \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)c}{2k+\beta+1} + \sum_{n=2}^{\infty} na_n |z|^{n-1}$$

$$\leq \frac{1}{r^2} + \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)c}{2k+\beta+1} + r\sum_{n=2}^{\infty} na_n$$

$$\leq \frac{1}{r^2} + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1} + \frac{2(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}r,$$

and

$$|f'(z)| \geq \left|\frac{-1}{z^2}\right| - \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)c}{2k+\beta+1} - \sum_{n=2}^{\infty} na_n |z|^{n-1}$$

$$\geq \frac{1}{r^2} - \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)c}{2k+\beta+1} - r\sum_{n=2}^{\infty} na_n$$

$$\geq \frac{1}{r^2} - \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1} - \frac{2(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{(3k+\beta+2)(2\lambda\alpha+\lambda-\alpha+1)}r.$$

Hence the result follows.

Next, we will show that the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$ is closed under convex linear combination.

Theorem 4. If

$$f_1(z) = \frac{1}{z} + \frac{(1-\beta)(2\lambda\alpha - 2\lambda + 2\alpha + 1)c}{2k+\beta+1}z, \tag{2.9}$$

and

$$f_n(z) = \frac{1}{z} + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}z + \sum_{n=2}^{\infty} \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}z^n \ (n \geq 2) \tag{2.10}$$

Then $f \in \Sigma_p(\lambda, \alpha, \beta, k, c)$ if and only if it expressed in the form

$$f(z) = \sum_{n=1}^{\infty} \mu_n f_n(z), \tag{2.11}$$

where $\mu_n \geq 0$ and $\sum_{n=1}^{\infty} \mu_n \leq 1$.

Proof. From (2.9), (2.10) and (2.11), we get

$$f(z) = \frac{1}{z} + \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}z + \sum_{n=2}^{\infty} \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)\mu_n}{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}z^n. \tag{2.13}$$

Since

$$\sum_{n=2}^{\infty} \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)\mu_n}{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]} \frac{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}$$

$$= \sum_{n=2}^{\infty} \mu_n = 1 - \mu_1 \leq 1,$$

it follows from Theorem 1 that the function $f \in \Sigma_p(\lambda, \alpha, \beta, k, c)$.
Conversely, suppose that $f \in \Sigma_p(\lambda, \alpha, \beta, k, c)$. Since

$$a_n \leq \frac{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}, \qquad n \geq 2.$$

Setting

$$\mu_n = \frac{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}{(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)} a_n \ ,$$

and

$$\mu_1 = 1 - \sum_{n=2}^{\infty} \mu_n < 1,$$

we get

$$f(z) = \sum_{n=1}^{\infty} \mu_n f_n(z). \tag{2.14}$$

This completes the proof of Theorem 4.

Theorem 5. The class $\Sigma_p(\lambda, \alpha, \beta, k, c)$ is closed under convex linear combination.

Proof. Suppose that each of the function $f_j(z)$ given by

$$f_j(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_{n,j} z^n, j = 1, 2$$

is in the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$. We need to prove that the function $h(z)$ given by

$$h(z) = \mu f_1(z) + (1-\mu) f_2(z) \ (0 \leq \mu \leq 1)$$

is also in the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$. Since

$$h(z) = \frac{1}{z} + \sum_{n=1}^{\infty} [\mu a_{n,1} + (1-\mu) a_{n,2}] z^n.$$

Consider

$$\begin{aligned}
&\sum_{n=1}^{\infty}[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1][\mu a_{n,1}+(1-\mu)a_{n,2}] \\
= &\ [\mu\sum_{n=1}^{\infty}[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]a_{n,1} \\
&+(1-\mu)\sum_{n=1}^{\infty}[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]a_{n,2}] \\
\leq &\ \mu(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)+(1-\mu)(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1) \\
\leq &\ (1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1).
\end{aligned}$$

Thus from Theorem 1, $h(z) \in \Sigma_p(\lambda, \alpha, \beta, k, c)$. Hence the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$ is closed under linear combination.

Now we determine the radii of meromorphically starlikeness and convexity of order δ for function in the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$.

Theorem 6. Let the function $f(z)$ defined by (1.10) be in the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$, then f is meromorphically starlike of order δ $(0 \leq \delta < 1)$ in 0¡$|z| < r_1(\lambda, \alpha, \beta, k, c, \delta)$, where $r_1(\lambda, \alpha, \beta, k, c, \delta)$, is the largest value for which

$$\frac{(3-\delta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}r^2+\frac{(n+2-\delta)(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n+\beta)+k(n+1)][(n-1)(n\lambda\alpha+\lambda-\alpha)+1]}r^{n+1} \leq (1-\delta)\,(n \geq 2).$$

The result is sharp for the function $f(z)$ is given by (2.4).

Proof. Suppose $f \in \Sigma_p(\lambda, \alpha, \beta, k, c)$, it is sufficient to show that

$$\left|1+\frac{zf'(z)}{f(z)}\right| \leq 1-\delta, \qquad (\text{for } 0 \leq \delta < 1, 0 < |z| < r_1(\lambda, \alpha, \beta, k, c, \delta)). \quad (2.15)$$

Replacing $f(z)$ and $zf'(z)$ with their equivalent series expansions in the left hand side of (2.15), we get

$$\left|1+\frac{zf'(z)}{f(z)}\right| = \left|\frac{-\frac{1}{z}+\frac{(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}z+\sum_{n=2}^{\infty}na_nz^n+\frac{1}{z}+\frac{(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}z+\sum_{n=2}^{\infty}a_nz^n}{\frac{1}{z}+\frac{(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}z+\sum_{n=2}^{\infty}a_nz^n}\right|. \quad (2.16)$$

Hence (2.16) holds true if

$$\frac{2\left(2\lambda\alpha-2\lambda+2\alpha+1\right)c}{2k+\beta+1}r^{2}+\sum_{n=2}^{\infty}\left(n+1\right)a_{n}r^{n+1}$$
$$\leq\ 1-\delta)\left[1-\frac{\left(2\lambda\alpha-2\lambda+2\alpha+1\right)c}{2k+\beta+1}r^{2}-\sum_{n=2}^{\infty}a_{n}r^{n+1}\right],$$

or,

$$\frac{\left(3-\delta\right)\left(2\lambda\alpha-2\lambda+2\alpha+1\right)c}{2k+\beta+1}r^{2}+\sum_{n=2}^{\infty}(n+2-\delta)a_{n}r^{n+1}\leq(1-\delta),$$

For each fixed r we choose the positive integer $n_0 = n_0\left(r\right)$ for which

$$\frac{(n+2-\delta)}{[(n+\beta)+k\left(n+1\right)][(n-1)\left(n\lambda\alpha+\lambda-\alpha\right)+1]}a_{n}r^{n+1}$$

is maximal. Then it follows that

$$\sum_{n=2}^{\infty}(n+2-\delta)a_{n}r^{n+1}\leq\frac{(n_0+2-\delta)(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n_0+\beta)+k(n_0+1)][(n_0-1)(n_0\lambda\alpha+\lambda-\alpha)+1]}r^{n_0+1}.$$

Then f is meromorphically starlike in $0 < |z| < r_1(\lambda, \alpha, \beta, k, c, \delta)$ of order δ provided that

$$\frac{(3-\delta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}r^{2}+\frac{(n_0+2-\delta)(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n_0+\beta)+k(n_0+1)][(n_0-1)(n_0\lambda\alpha+\lambda-\alpha)+1]}r^{n_0+1}\leq(1-\delta). \quad (2.17)$$

We find the value $r_0 = r_0(\lambda, \alpha, \beta, k, c, \delta)$ and the corresponding integer $n_0\left(r_0\right)$ so that

$$\frac{(3-\delta)(2\lambda\alpha-2\lambda+2\alpha+1)c}{2k+\beta+1}r_0^{2}+\frac{(n_0+2-\delta)(1-\beta)(2\lambda\alpha-2\lambda+2\alpha+1)(1-c)}{[(n_0+\beta)+k(n_0+1)][(n_0-1)(n_0\lambda\alpha+\lambda-\alpha)+1]}r_0^{n_0+1}=(1-\delta). \quad (2.18)$$

Then this value r_0 is the radius of meromorphically starlikeness for functions $f(z)$ belonging to the class $\Sigma_p(\lambda, \alpha, \beta, k, c)$. This completes the proof of Theorem 6.

3. **Open problems**

The authors suggest to study the above problem for analytic p-valent meromorphic functions with fixed a_p coefficient, also discusses the neighborhood and partial sum problems for the new class

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