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Weighted Value Sharing Results for Meromorphic Function Concerning q-shift Differential Difference Polynomials

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Abstract

In this paper, we deal with the uniqueness problems on meromorphic functions of certain type of q-shift differential-difference polynomials of zero order with the aid of weighted sharing values. Moreover, the results of this paper improve and extend some earlier results, which were obtained individually by Dyavanal, Xu and Cao, etc.

Keywords: *Differential-Difference polynomial, Meromorphic functions, q-shift, Uniqueness, Weighted sharing, Zero order.*

2020 Mathematical Subject Classification: 30D35.

1 Background

In this paper, we use standard notation and fundamental results of the Nevanlinna theory([3], [16], [17]). In the uniqueness theory of meromorphic functions, we study conditions under which there exists essentially only one function satisfying the given hypothesis and hence how to uniquely determine a meromorphic function is interesting. Nevanlinna himself proved that any nonconstant meromorphic function can be uniquely determined by five values. In other words, if two nonconstant meromorphic functions Φ and Ψ take same five values at the same points, then $\Phi \equiv \Psi$. There has been an increasing interest in uniqueness theory of q-shift polynomials can be seen in ([11], [10], [4]). Many articles focused on uniqueness of entire or meromorphic functions([12], [2], [18], [8], [5], [15]). The notation of weighted sharing are explained in ([6], [7], [1]).

In this paper, by introducing the notion of multiplicity we establish the uniqueness result for q-shift differential-difference polynomial of the form $[\Phi^n P(\Phi(qz+\eta))]^{(k)}$. Now by taking $q=1$ and $\eta=0$ then the polynomial $[\Phi^n P(\Phi(qz+\eta))]^{(k)}$ reduces to the form $[\Phi^n P(\Phi(z))]^{(k)}$ which is same as considered by Xu et al.([14]). Hence the polynomial $[\Phi^n P(\Phi(qz+\eta))]^{(k)}$ is of more general form. We also relax the nature of sharing, reduce the lower bound of n and obtain the following results.

Theorem 1. Let Φ and Ψ be two non-constant meromorphic functions, whose zeros and poles are of multiplicities atleast s, where s is a positive integer. Let $P(\Phi) = a_m\Phi^m + a_{m-1}\Phi^{m-1} + ... + a_1\Phi + a_0$, $(a_m \neq 0)$, and $a_i(i = 0, 1, ..., m)$ is nonzero coefficient, and let n, k, m be three positive integers. If $[\Phi^n P(\Phi(qz+\eta))]^{(k)}$ and $[\Psi^n P(\Psi(qz+\eta))]^{(k)}$ share $(1, l)$ and one of the following conditions holds:
(i) $l \geq 2$ and $s(n+m) > 3k+10$,
(ii) $l = 1$ and $s(n+m) > 5k+13$,
(iii) $l = 0$ and $s(n+m) > 9k+16$,
then either $\Phi = t\Psi$ for a constant t such that $t^d = 1$, where $d = (n+m, ...n+m-i, ..., n)$, $a_{m-i} \neq 0$ for some $i = 0, 1...m$, or Φ and Ψ satisfy the algebraic equation $R(\Phi, \Psi) \equiv 0$, where $R(\omega_1, \omega_2) = \omega_1^n P(\omega_1) - \omega_2^n P(\omega_2)$.

Theorem 2. Let Φ and Ψ be two non-constant entire functions, whose zeros and poles are of multiplicities atleast s, where s is a positive integer. Let $P(\Phi) = a_m\Phi^m + a_{m-1}\Phi^{m-1} + ... + a_1\Phi + a_0$, $(a_m \neq 0)$, and $a_i(i = 0, 1, ..., m)$ is nonzero coefficient, and let n, k, m be three positive integers. If $[\Phi^n P(\Phi(qz+\eta))]^{(k)}$ and $[\Psi^n P(\Psi(qz+\eta))]^{(k)}$ share $(1, l)$ and one of the following conditions holds:
(i) $l \geq 2$ and $s(n+m) > 3k+5$,
(ii) $l = 1$ and $s(n+m) > 4k+6$,
(iii) $l = 0$ and $s(n+m) > 5k+8$,
then either $\Phi = t\Psi$ for a constant t such that $t^d = 1$, where $d = (n+m, ...n+m-i, ..., n)$, $a_{m-i} \neq 0$ for some $i = 0, 1...m$, or Φ and Ψ satisfy the algebraic equation $R(f, g) \equiv 0$, where $R(\omega_1, \omega_2) = \omega_1^n P(\omega_1) - \omega_2^n P(\omega_2)$.

Remark. In Theorem 2, giving specific values for s, we get the following interesting cases:
(i) If $s = 1$, then for $l \geq 2$ we get $n > 3k+5-m$, for $l = 1$ we get $n > 4k+6-m$ and for $l = 0$ we get $n > 5k+8-m$.
(ii) If $s = 2$, then for $l \geq 2$ we get $n > \frac{3k+5}{2} - m$, for $l = 1$ we get $n > 2k+3-m$

and for $l = 0$ we get $n > \frac{5k+8}{2} - m$.

We conclude that if Φ and Ψ have zeros and poles of higher order multiplicity, then we can reduce the value of n.

2 Some Lemmas

Lemma 1[3]. Let Φ be a nonconstant meromorphic function, let k be a positive integer, and let c be a nonzero finite complex number. Then

$$\begin{aligned} T(r,\Phi) &\leq \overline{N}(r,\Phi) + N\left(r,\frac{1}{\Phi}\right) + N\left(r,\frac{1}{\Phi^{(k)}-c}\right) - N\left(r,\frac{1}{\Phi^{(k+1)}}\right) + S(r,\Phi), \\ &\leq \overline{N}(r,\Phi) + N_{k+1}\left(r,\frac{1}{\Phi}\right) + \overline{N}\left(r,\frac{1}{\Phi^{(k)}-c}\right) - N_0\left(r,\frac{1}{\Phi^{(k+1)}}\right) \\ &+ S(r,\Phi). \end{aligned}$$

where $N_0\left(r,\frac{1}{\Phi^{(k+1)}}\right)$ is the counting function which only counts those points such that $\Phi^{(k+1)} = 0$ but $\Phi(\Phi^{(k)} - c) \neq 0$.

Lemma 2[16]. Let Φ be a nonconstant meromorphic function and $P(\Phi) = a_0 + a_1\Phi + ... + a_n\Phi^n$,where a_0, a_1,...,a_n are constants and $a_n \neq 0$. Then

$$T(r, P(\Phi)) = nT(r,\Phi) + S(r,\Phi).$$

Lemma 3[13]. Let $\Phi(z)$ be a nonconstant meromorphic function of zero order and let q, η be two nonzero complex constants. Then on a set of lower logarithmic density 1, we have

$$T(r,\Phi(qz+\eta)) = T(r,\Phi) + S(r,\Phi). \tag{1}$$

Lemma 4[13]. Let $\Phi(z)$ be a nonconstant meromorphic function of zero order and let q, η be two nonzero complex constants. Then on a set of lower logarithmic density 1, we have

$$N(r,\Phi(qz+\eta)) = N(r,\Phi) + S(r,\Phi), \tag{2}$$

$$N\left(r,\frac{1}{\Phi(qz+\eta)}\right) = N\left(r,\frac{1}{\Phi}\right) + S(r,\Phi). \tag{3}$$

Lemma 5[13]. Let $\Phi(z)$ be a nonconstant meromorphic function of zero order and let q be a nonzero complex number. Then on a set of lower logarithmic density 1, we have

$$m\left(r,\frac{\Phi(qz+\eta)}{\Phi(z)}\right) = S(r,\Phi). \tag{4}$$

Lemma 6([5], [19]). Let Φ be a nonconstant meromorphic function and k be a positive integer, then

$$\begin{aligned} N_p\left(r,\frac{1}{\Phi^{(k)}}\right) &\leq N_{p+k}\left(r,\frac{1}{\Phi}\right)+k\overline{N}(r,\Phi)+S(r,\Phi),\\ &\leq (p+k)\overline{N}\left(r,\frac{1}{\Phi}\right)+k\overline{N}(r,\Phi)+S(r,\Phi). \end{aligned}$$

This lemma can be obtained immediately from the proof of Lemma 2.3 in [5] which is the case $p = 2$.

Lemma 7[20]. Let Φ and Ψ be two nonconstant meromorphic functions. If Φ and Ψ share 1 IM, then

$$\overline{N}_L\left(r,\frac{1}{\Phi-1}\right)\leq \overline{N}\left(r,\frac{1}{\Phi}\right)+\overline{N}(r,\Phi)+S(r,\Phi). \tag{5}$$

Lemma 8[14]. Let Φ and Ψ be two nonconstant entire functions, and let k be a positive integer. If $\Phi^{(k)}$ and $\Psi^{(k)}$ share $(1,l)$ $(l=0,1,2)$, then

(i) If $l = 0$,

$$\Theta(0,\Phi)+\delta_k(0,\Phi)+\delta_{k+1}(0,\Phi)+\delta_{k+1}(0,\Psi)+\delta_{k+2}(0,\Phi)+\delta_{k+2}(0,\Psi)>5,$$

then either $\Phi^{(k)}\Psi^{(k)}=1$ or $\Phi\equiv\Psi$;

(ii) If $l = 1$,

$$\frac{1}{2}\left[\Theta(0,\Phi)+\delta_k(0,\Phi)+\delta_{k+2}(0,\Phi)\right]+\delta_{k+1}(0,\Phi)+\delta_{k+1}(0,\Psi)+\Theta(0,\Psi)+\delta_k(0,\Psi)>\frac{9}{2},$$

then either $\Phi^{(k)}\Psi^{(k)}=1$ or $\Phi\equiv\Psi$;

(iii) If $l = 2$,

$$\Theta(0,\Phi)+\delta_k(0,\Phi)+\delta_{k+1}(0,\Phi)+\delta_{k+2}(0,\Psi)>3,$$

then either $\Phi^{(k)}\Psi^{(k)}=1$ or $\Phi\equiv\Psi$.

Lemma 9[12]. Let Φ and Ψ be two nonconstant meromorphic functions, $k(\geq 1)$ and l (≥ 0) be integers. If $\Phi^{(k)}$ and $\Psi^{(k)}$ share $(1,l)$ $(l=0,1,2)$, then
(i) If $l\geq 2$,
$(k+2)\Theta(\infty,\Phi)+2\Theta(\infty,\Psi)+\Theta(0,\Phi)+\Theta(0,\Psi)+\delta_{k+1}(0,\Phi)+\delta_{k+1}(0,\Psi)>k+7$,
then either $\Phi^{(k)}\Psi^{(k)}=1$ or $\Phi\equiv\Psi$;

(ii) If $l = 1$,
$(2k+3)\Theta(\infty,\Phi) + 2\Theta(\infty,\Psi) + \Theta(0,\Phi) + \Theta(0,\Psi) + \delta_{k+1}(0,\Phi) + \delta_{k+1}(0,\Psi) + \delta_{k+2}(0,\Phi) > 2k+9$, then either $\Phi^{(k)}\Psi^{(k)} = 1$ or $\Phi \equiv \Psi$;

(iii) If $l = 0$,
$(2k+3)\Theta(\infty,\Phi)+(2k+4)\Theta(\infty,\Psi)+\Theta(0,\Phi)+\Theta(0,\Psi)+2\delta_{k+1}(0,\Phi)+3\delta_{k+1}(0,\Psi) > 4k+13$, then either $\Phi^{(k)}\Psi^{(k)} = 1$ or $\Phi \equiv \Psi$.

Lemma 10. Let Φ and Ψ be two non-constant meromorphic functions, and let $n(\geq 1)$, $k(\geq 1)$ and $m(\geq 1)$ be integers. Then

$$[\Phi^n P(\Phi(qz+\eta))]^{(k)}[\Psi^n P(\Psi)(qz+\eta))]^{(k)} \neq 1.$$

Proof. Let

$$[\Phi^n P(\Phi(qz+\eta))]^{(k)}[\Psi^n P(\Psi(qz+\eta))]^{(k)} \equiv 1. \tag{6}$$

Let z_0 be a zero of Φ of order p_0. From equation (6) we get z_0 is a pole of Ψ. Suppose that z_0 is a pole of Ψ of order q_0. Again by equation (6), we obtain $np_0 - k = nq_0 + mq_0 + k$,

i.e., $n(p_0 - q_0) = mq_0 + 2k$.

which implies that $q_0 \geq \frac{n-2k}{m}$ and so we have $p_0 \geq \frac{n+m-2k}{m}$.

Let z_1 be a zero of $\Phi - 1$ of order p_1, then z_1 is a zero of $[\Phi^n P(\Phi)]^{(k)}$ of order $p_1 - k$. Therefore from equation (6) we obtain $p_1 - k = nq_1 + mq_1 + k$,

i.e., $p_1 \geq (n+m)s + 2k$.

Let z_2 be a zero of Φ' of order p_2 that is not a zero of $\Phi P(\Phi)$, then from equation (6) z_2 is a pole of Ψ of order q_2. Again by equation (6) we get $p_2 - (k-1) = nq_2 + mq_2 + k$,

i.e., $p_2 \geq (n+m)s + 2k - 1$.

In the same manner as above, we have similar results for the zeros of $[\Psi^n P(\Psi)]^{(k)}$.

On the other hand, suppose that z_3 is a pole of f. From equation (6), we find that z_3 is the zero of $[\Psi^n P(\Psi)]^{(k)}$.

Thus

$$\begin{aligned}\overline{N}(r,\Phi) &\le \overline{N}\left(r,\frac{1}{\Psi}\right)+\overline{N}\left(r,\frac{1}{\Psi-1}\right)+\overline{N}\left(r,\frac{1}{\Psi'}\right)\\ &\le \frac{1}{p_0}N\left(r,\frac{1}{\Psi}\right)+\frac{1}{p_1}N\left(r,\frac{1}{\Psi-1}\right)+\frac{1}{p_2}N\left(r,\frac{1}{\Psi'}\right)\\ &\le \left[\frac{m}{n+m-2k}+\frac{1}{(n+m)s+2k}+\frac{2}{(n+m)s+2k-1}\right]T(r,\Psi)+S(r,\Psi).\end{aligned} \tag{7}$$

By the second fundamental theorem and equation (7), we have

$$\begin{aligned}T(r,\Psi) &\le \overline{N}\left(r,\frac{1}{\Phi}\right)+\overline{N}\left(r,\frac{1}{\Phi-1}\right)+\overline{N}(r,\Phi)\\ &\le \frac{m}{n+m-2k}N\left(r,\frac{1}{\Phi}\right)+\frac{1}{(n+m)s+2k}N\left(r,\frac{1}{\Phi-1}\right)\\ &+ \left[\frac{m}{n+m-2k}+\frac{1}{(n+m)s+2k}+\frac{2}{(n+m)s+2k-1}\right]T(r,\Psi)\\ &+ S(r,\Psi)+S(r,\Phi).\end{aligned}$$

$$\begin{aligned}T(r,\Phi) \le& \left[\frac{m}{n+m-2k}+\frac{1}{(n+m)s+2k}\right]T(r,\Phi)\\ &+\left[\frac{m}{n+m-2k}+\frac{1}{(n+m)s+2k}+\frac{2}{(n+m)s+2k-1}\right]T(r,\Psi)\\ &+S(r,\Psi)+S(r,\Phi).\end{aligned} \tag{8}$$

Similarly, we have

$$\begin{aligned}T(r,\Psi) \le& \left[\frac{m}{n+m-2k}+\frac{1}{(n+m)s+2k}\right]T(r,\Psi)\\ &+\left[\frac{m}{n+m-2k}+\frac{1}{(n+m)s+2k}+\frac{2}{(n+m)s+2k-1}\right]T(r,\Phi)\\ &+S(r,\Psi)+S(r,\Phi).\end{aligned} \tag{9}$$

Adding equations (8) and (9) we get

$$\begin{aligned}T(r,\Phi)+T(r,\Psi) &\le \Big[\frac{2m}{n+m-2k}+\frac{2}{(n+m)s+2k}\\ &+ \frac{2}{(n+m)s+2k-1}\Big]\{T(r,\Phi)+T(r,\Psi)\}+S(r,\Psi)\\ &+ S(r,\Phi).\end{aligned}$$

which is a contradiction. Thus the lemma is proved.

3 Proofs of the Theorems

In this section we present the proofs of the main results.

Proof of Theorem 1.

Let $F = \Phi^n P\left(\Phi(qz+\eta)\right)$ and $G = \Psi^n P\left(\Psi(qz+\eta)\right)$.

Consider

$$\overline{N}\left(r, \frac{1}{F}\right) = \overline{N}\left(r, \frac{1}{\Phi^n P(\Phi)}\right) \leq \frac{1}{s(n+m)} N\left(r, \frac{1}{F}\right) \leq \frac{2}{s(n+m)}[T(r,F)+O(1)].$$

$$\Theta(0,F) = 1 - \limsup_{r\to\infty} \frac{\overline{N}\left(r, \frac{1}{F}\right)}{T(r,F)} \geq 1 - \frac{2}{s(n+m)}. \tag{10}$$

Similarly,

$$\Theta(0,G) \geq 1 - \frac{2}{s(n+m)}, \tag{11}$$

and

$$\Theta(\infty,F) = 1 - \limsup_{r\to\infty} \frac{\overline{N}\left(r, F\right)}{T(r,F)} \geq 1 - \frac{1}{s(n+m)}. \tag{12}$$

Similarly,

$$\Theta(\infty,G) \geq 1 - \frac{1}{s(n+m)}. \tag{13}$$

Consider

$$N_{k+1}\left(r, \frac{1}{F}\right) = N_{k+1}\left(r, \frac{1}{\Phi^n P(\Phi)}\right) = (k+1)\overline{N}\left(r, \frac{1}{\Phi^n P(\Phi)}\right) \leq \frac{(k+1)}{s(n+m)}[T(r,F)+O(1)].$$

Next, we have

$$\delta_{k+1}(0,F) = 1 - \limsup_{r\to\infty} \frac{N_{k+1}\left(r, \frac{1}{F}\right)}{T(r,F)} \geq 1 - \frac{(k+1)}{s(n+m)}. \tag{14}$$

Similarly,

$$\delta_{k+1}(0,G) \geq 1 - \frac{(k+1)}{s(n+m)}. \tag{15}$$

Case (i) If $l \geq 2$ and from (10) to (15) and also from Lemma 6, we get

$$\begin{aligned}\Delta_1 &= (k+2)\Theta(\infty,\Phi) + 2\Theta(\infty,\Psi) + \Theta(0,\Phi) + \Theta(0,\Psi) + \delta_{k+1}(0,\Phi) + \delta_{k+1}(0,\Psi) \\ &> (k+8) - \frac{3k+10}{s(n+m)}.\end{aligned}$$

Since $s(n+m) > 3k+10$, we get $\Delta_1 > k+7$.

Therefore, by Lemma 6, we deduce that either $F^{(k)}G^{(k)} \equiv 1$ or $F \equiv G$.

If $F^{(k)}G^{(k)} \equiv 1$, that is

$$[\Phi^n(a_m\Phi^m+a_{m-1}\Phi^{m-1}+...+a_1\Phi+a_0)]^{(k)}[\Psi^n(a_m\Psi^m+a_{m-1}\Psi^{m-1}+...+a_1\Psi+a_0)]^{(k)} \equiv 1, \tag{16}$$

then by Lemma 7 we can get a contradiction.

Hence, we deduce that $F \equiv G$, that is

$$\Phi^n(a_m\Phi^m+a_{m-1}\Phi^{m-1}+...+a_1\Phi+a_0) = \Psi^n(a_m\Psi^m+a_{m-1}\Psi^{m-1}+...+a_1\Psi+a_0). \tag{17}$$

Let $h = \frac{\Phi}{\Psi}$. If h is a constant, then substituting $\Phi = \Psi h$ in (17) we obtain

$$a_m\Psi^{n+m}(h^{n+m}-1) + a_{m-1}\Psi^{n+m-1}(h^{n+m-1}-1) + ... + a_0\Psi^n(h^n-1) = 0,$$

which implies $h^d = 1$, where $d = (n+m, ..., n+m-i, ...n)$, $a_{m-1} \neq 0$ for some $i = 0, 1, ...m$. Thus $\Phi \equiv t\Psi$ for a constant t such that $t^d = 1$, where $d = (n+m, ..., n+m-i, ...n)$, $a_{m-i} \neq 0$ for some $i = 0, 1, ...m$. If h is not a constant, then we know from (17) that Φ and Ψ satisfy the algebraic equation $R(\Phi, \Psi) = 0$, where $R(\omega_1, \omega_2) = \omega_1^n P(\omega_1) - \omega_2^n P(\omega_2)$.

Case (ii) If $l = 1$ and from (10) to (15) and also from Lemma 6, we get

$$\begin{aligned}\Delta_2 &= (2k+3)\Theta(\infty,\Phi) + 2\Theta(\infty,\Psi) + \Theta(0,\Phi) + \Theta(0,\Psi) + \delta_{k+1}(0,\Phi)\\ &+ \delta_{k+1}(0,\Psi) + \delta_{k+2}(0,\Phi)\\ &> (2k+10) - \frac{5k+13}{s(n+m)}.\end{aligned}$$

Since $s(n+m) > 5k+13$, we get $\Delta_2 > 2k+9$.

By continuing as in case(i), we get case(ii).

Case (iii) If $l = 0$ and from (10) to (15) and also from Lemma 6, we get

$$\begin{aligned}\Delta_3 &= (2k+3)\Theta(\infty,\Phi) + (2k+4)\Theta(\infty,\Psi) + \Theta(0,\Phi) + \Theta(0,\Psi) + 2\delta_{k+1}(0,\Phi)\\ &+ 3\delta_{k+1}(0,\Psi)\\ &> (4k+14) - \frac{9k+16}{s(n+m)}.\end{aligned}$$

Since $s(n+m) > 9k+16$, we get $\Delta_2 > 4k+13$.

By continuing as in case (i), we get case (iii).

This completes the proof of Theorem 1 .

Proof of Theorem 2.

Since Φ and Ψ are entire functions we have $\overline{N}(r,\Phi)=\overline{N}(r,\Psi)=0$. Proceeding as in the proof of Theorem 1 we can easily prove Theorem 2 .

Now we present the following corollaries of Theorem 1 and Theorem 2 .

Corollary 1. Let Φ and Ψ be two nonconstant meromorphic functions, whose zeros and poles are of multiplicities atleast s, where s is a positive integer. Let $P(\Phi)=a_m\Phi^m+a_{m-1}\Phi^{m-1}+...+a_1\Phi+a_0$, $(a_m\neq 0)$, and $a_i(i=0,1,...,m)$ is nonzero coefficient, and let n, m be two positive integers. If $\Phi^nP\left(\Phi(qz+\eta)\right)$ and $\Psi^nP\left(\Psi(qz+\eta)\right)$ share $(1,l)$ and one of the following conditions holds:
(i) $l\geq 2$ and $s(n+m)>10$,
(ii) $l=1$ and $s(n+m)>13$,
(iii) $l=0$ and $s(n+m)>16$,
then either $\Phi=t\Psi$ for a constant t such that $t^d=1$, where $d=(n+m,...n+m-i,...,n)$, $a_{m-i}\neq 0$ for some $i=0,1...m$, or Φ and Ψ satisfy the algebraic equation $R(\Phi,\Psi)\equiv 0$, where $R(\omega_1,\omega_2)=\omega_1^nP(\omega_1)-\omega_2^nP(\omega_2)$.

Corollary 2. Let Φ and Ψ be two nonconstant entire functions, whose zeros and poles are of multiplicities atleast s, where s is a positive integer. Let $P(\Phi)=a_m\Phi^m+a_{m-1}\Phi^{m-1}+...+a_1\Phi+a_0$, $(a_m\neq 0)$, and $a_i(i=0,1,...,m)$ is nonzero coefficient, and let n, m be two positive integers. If $\Phi^nP\left(\Phi(qz+\eta)\right)$ and $\Psi^nP\left(\Psi(qz+\eta)\right)$ share $(1,l)$ and one of the following conditions holds:
(i) $l\geq 2$ and $s(n+m)>5$,
(ii) $l=1$ and $s(n+m)>6$,
(iii) $l=0$ and $s(n+m)>8$,
then either $\Phi=t\Psi$ for a constant t such that $t^d=1$, where $d=(n+m,...n+m-i,...,n)$, $a_{m-i}\neq 0$ for some $i=0,1...m$, or Φ and Ψ satisfy the algebraic equation $R(f,g)\equiv 0$, where $R(\omega_1,\omega_2)=\omega_1^nP(\omega_1)-\omega_2^nP(\omega_2)$.

4 Conclusions

In this paper, by introducing notion of multiplicity and considering more general forms of polynomial we prove two theorems which extend and improve the results due to ([2], [14], [18]). Proving the results without complicated calculations is a feature of mathematical elegance of this paper.

5 Open Problem

1. What can be said if we consider the difference-differential polynomials of the form $\left[\Phi^n P(\Phi) \prod_{j=1}^{d} \Phi(qz+\eta)^{v_j}\right]^{(k)}$, where $P(\Phi) = a_m\Phi^m + a_{m-1}\Phi^{m-1} + ... + a_1\Phi + a_0$, $(a_m \neq 0)$, and $a_i(i = 0, 1, ..., m)$ is nonzero coefficient.

2. Whether it is possible to replace the weighted sharing value by small function.

3. Is it possible to reduce the condition of the theorem.

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