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Some Inequalities for Meromorphic Multivalent Functions Associated with Mittag-Leffler Function

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Abstract

The purpose of this paper is to prove some differential inequalities for meromorphic multivalent functions by using a new operator associated with Mittag-Leffler function..

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1 Introduction

Denote by Σ_p the class of meromorphic multivalent functions of the form:

$$f(z) = \frac{1}{z^p} + \sum_{n=1-p}^{\infty} a_n z^n \ , \qquad (1)$$

which are analytic in $\mathbb{U}^* = \{z : z \in \mathbb{C} \text{ and } 0 < |z| < 1\} = \mathbb{U}\backslash\{0\}$.

The Mittag-Leffler function $E_\alpha(z)$ (z $\in \mathbb{C}$) ([7], [8] and see also [3-5]) is defined by

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(n\alpha + 1)} z^n, \alpha \in \mathbb{C}, Re(\alpha) > 0.$$

For $\alpha, \beta, \gamma \in \mathbb{C}$, $Re(\alpha) > max0, Re(k) - 1$ and $Re(k) > 0)$, Srivastava and Tomovski [9] generalized Mittag-Leffler function and introduced the function

$$E_{\alpha,\beta}^{\gamma,k}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nk}}{\Gamma(n\alpha + \beta)n!} z^n \tag{2}$$

and proved that it is an entire function in the complex z-plane, where

$$(\gamma)_\theta = \frac{\Gamma(\gamma+\theta)}{\Gamma(\gamma)} \begin{cases} 1, & \theta = 0 \\ \gamma(\gamma+1)...(\gamma+\theta-1), & \theta \neq 0 \end{cases} .$$

Using the function $E_{\alpha,\beta}^{\gamma,k}(z)$, let

$$\begin{aligned} \mathcal{M}_{p,\alpha,\beta}^{\gamma,k}(z) &= z^{-p}\Gamma(\beta)E_{\alpha,\beta}^{\gamma,k}(z) \\ &= z^{-p} + \sum_{n=1-p}^{\infty} \frac{\Gamma(\beta)\Gamma[\gamma + (n+p)k]}{\Gamma(\gamma)\Gamma[\beta + (n+p)\alpha]\Gamma(n+p)} z^n, \end{aligned}$$

$$Re\alpha = 0 \ when \ Rek = 1 \ with \ \beta \neq 0. \tag{3}$$

For $f(z) \in \Sigma_p$, we define the operator

$$\begin{aligned} \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z) &= \mathcal{M}_{p,\alpha,\beta}^{\gamma,k}(z) * f(z) \\ &= z^{-p} + \sum_{n=1-p}^{\infty} \frac{\Gamma(\beta)\Gamma[\gamma + (n+p)k]}{\Gamma(\gamma)\Gamma[\beta + (n+p)\alpha]\Gamma(n+p)} a_n z^n. \end{aligned} \tag{4}$$

From (4) it is easy to have

$$\mathcal{H}_{p,0,\beta}^{1,1} f(z) = pf(z) + zf'(z) + z^{-p},$$

$$kz(\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z))' = \gamma \mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z) - (\gamma + pk)\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z) \tag{5}$$

and

$$\alpha z \left(\mathcal{H}_{p,\alpha,\beta+1}^{\gamma,k} f(z) \right)' = \beta \ \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z) - (p\alpha + \beta) \ \mathcal{H}_{p,\alpha,\beta+1}^{\gamma,k} f(z), \alpha \neq 0. \tag{6}$$

In the present paper using the operator $\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)$, we investigate some inequalties for meromorphic univalent functions.

Definition 1. Let H be the set of complex valued functions $h(r,s,t) : \mathbb{C}^3 \to \mathbb{C}$ such that:

i) is continuous in a domain $D \subset \mathbb{C}^3$

ii) $(1,1,1) \in D$, $|h(1,1,1)| < 1$

iii)

$$\left|h\left(e^{i\theta},(1+\frac{k}{\gamma}\zeta)e^{i\theta},k^2\frac{\frac{\gamma(\gamma+1)}{k^2}e^{ii\theta}+\frac{2\gamma+k+1}{k}\zeta e^{i\theta}+L}{\gamma(\gamma+1)}\right)\right|>1, \quad (7)$$

whenever

$$\left(e^{i\theta},(1+\frac{k}{\gamma}\zeta)e^{i\theta},k^2\frac{\frac{\gamma(\gamma+1)}{k^2}e^{ii\theta}+\frac{2\gamma+k+1}{k}\zeta e^{i\theta}+L}{\gamma(\gamma+1)}\right)\in D$$

with $Ree^{-i\theta}L\geq\zeta(\zeta-1),\zeta\geq 1$ and θ real.

Definition 2. Let K be the set of complex valued functions $k(r,s,t):\mathbb{C}^3\to\mathbb{C}$ such that:

i) is continuous in a domain $D\subset\mathbb{C}^3$

ii) $(1,1,1)\in D$, $|h(1,1,1)|<1$

iii)

$$\left|h\left(e^{i\theta},(1+\frac{\alpha}{\beta-1}\zeta)e^{i\theta},\alpha^2\frac{\frac{(\beta-1)(\beta-2)}{\alpha^2}e^{ii\theta}+\frac{\alpha+2\beta-3}{\alpha}\zeta e^{i\theta}+L}{(\beta-1)(\beta-2)}\right)\right|>1, \quad (8)$$

whenever

$$\left(e^{i\theta},(1+\frac{\alpha}{\beta-1}\zeta)e^{i\theta},\alpha^2\frac{\frac{(\beta-1)(\beta-2)}{\alpha^2}e^{ii\theta}+\frac{\alpha+2\beta-3}{\alpha}\zeta e^{i\theta}+L}{(\beta-1)(\beta-2)}\right)\in D$$

with $Ree^{-i\theta}L\geq\zeta(\zeta-1),\zeta\geq 1$ and θ real.

Definition 3. Let M be the set of complex valued functions $k(r,s,t):\mathbb{C}^3\to\mathbb{C}$ such that:

i) is continuous in a domain $D\subset\mathbb{C}^3$

ii) $(1,1,1)\in D$, $|h(1,1,1)|<1$

iii)

$$\left|h\left(e^{i\theta},\frac{\gamma e^{i\theta}+k\zeta+1}{\gamma+1},\frac{1}{(\gamma+2)}\left\{\gamma e^{i\theta}+k\zeta+2+k^2\frac{\frac{\gamma}{k}\zeta e^{i\theta}+\zeta-\zeta^2+L}{\gamma e^{i\theta}+k\zeta+1}\right\}\right)\right|>1, \quad (9)$$

whenever

$$\left(e^{i\theta},\frac{\gamma e^{i\theta}+k\zeta+1}{\gamma+1},\frac{1}{(\gamma+2)}\left\{\gamma e^{i\theta}+k\zeta+2+k^2\frac{\frac{\gamma}{k}\zeta e^{i\theta}+\zeta-\zeta^2+L}{\gamma e^{i\theta}+k\zeta+1}\right\}\right)\in D$$

with $ReL\geq\zeta(\zeta-1),\zeta\geq 1$ and θ real.

Definition 4. Let N be the set of complex valued functions $k(r,s,t):\mathbb{C}^3\to\mathbb{C}$ such that:

i) is continuous in a domain $D\subset\mathbb{C}^3$

ii) $(1,1,1)\in D$, $|h(1,1,1)|<1$

iii)

$$\left| h\left(e^{i\theta}, \frac{\beta e^{i\theta}+\alpha\zeta-1}{\beta-1}, \frac{1}{(\beta-2)}\left\{ \beta e^{i\theta}+\alpha\zeta-2+\alpha^{2\frac{\frac{\beta}{\alpha}\zeta e^{i\theta}+\zeta-\zeta^{2}+L}{\beta e^{i\theta}+\alpha\zeta+1}} \right\} \right) \right| > 1, \tag{10}$$

whenever

$$\left(e^{i\theta}, \frac{\beta e^{i\theta}+\alpha\zeta-1}{\beta-1}, \frac{1}{(\beta-2)}\left\{ \beta e^{i\theta}+\alpha\zeta-2+\alpha^{2\frac{\frac{\beta}{\alpha}\zeta e^{i\theta}+\zeta-\zeta^{2}+L}{\beta e^{i\theta}+\alpha\zeta+1}} \right\} \right) \in D$$

with $ReL \geq \zeta(\zeta-1), \zeta \geq 1$ and θ real.

2 Main Results

Unless otherwise maintained we assume that $L \geq \zeta(\zeta-1)$, $\zeta \geq 1, \alpha, k \neq 0, \beta \neq 0, 1, 2, \gamma \neq -1, -2$ and θ real.

To prove our main results, we need the following lemma.

Lemma 1[6]. *Let* $w(z) = a + a_{\tau}z^{\tau} + a_{\tau+1}z^{\tau+1}...$ *be analytic in* $\mathbb{U}$ *with with* $w(z) \neq a$ *and* $\tau \geq 1$. *If* $z_0 = r_0 e^{i\theta}(0 < r_0 < 1)$ *and* $|w(z_0)| = \max_{|z| \leq |z_0|} |w(z)|$. *Then*

$$z_0 w'(z_0) = \zeta w(z_0) \tag{11}$$

and

$$Re\left(1 + \frac{z_0 w''(z_0)}{w'(z_0)}\right) \geq \zeta, \tag{12}$$

where ζ *is a real number and*

$$\zeta \geq n\frac{|w(z_0)-a|^2}{|w(z_0)|^2 - |a|^2} \geq \tau\frac{|w(z_0)|-|a|}{|w(z_0)|+|a|}.$$

Theorem 1. *Let* $h(r,s,t) \in H$ *and* $f(z) \in \Sigma_p$ *satisfy:*

$$\left(z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z) \right) \in D \subset \mathbb{C}^3 \tag{13}$$

and

$$\left| h\left(z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z), z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z) \right) \right| < 1. \tag{14}$$

Then

$$\left| z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z) \right| < 1.$$

Proof. For $f \in \Sigma_p$, let

$$w(z) = z^p \mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z),$$

then $w(z)$ is either analytic or meromorphic in $\mathbb{U}$, $w(0) = 1$ and $w(z) \neq 1$. Using (5) we have

$$z^p \mathcal{H}^{\gamma+1,k}_{p,\alpha,\beta} f(z) = w(z) + \frac{k}{\gamma} z w'(z)$$

and

$$z^p \mathcal{H}^{\gamma+2,k}_{p,\alpha,\beta} f(z) = \frac{k^2}{\gamma(\gamma+1)} \left\{ \frac{\gamma(\gamma+1)}{k^2} w(z) + \frac{k+2\gamma+1}{k} z w'(z) + z^2 w''(z) \right\}.$$

Claim that $|w(z)| < 1, z \in \mathbb{U}$. If it is not, then there exists a point $z_0 \in \mathbb{U}$, $\underset{|z|<|z_0|}{max} |w(z)| = |w(z)| = 1$. Taking $w(z_0) = e^{i\theta}$ and by Lemma 1 with $a = \tau = 1$, we see that

$$z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z_0) = w(z_0) = e^{i\theta},$$

$$z^p \mathcal{H}^{\gamma+1,k}_{p,\alpha,\beta} f(z_0) = e^{i\theta} + \frac{k}{\gamma} \zeta e^{i\theta}$$

and

$$z^p \mathcal{H}^{\gamma+2,k}_{p,\alpha,\beta} f(z_0) = \frac{k^2}{\gamma(\gamma+1)} \left\{ \frac{\gamma(\gamma+1)}{k^2} e^{i\theta} + \frac{k+2\gamma+1}{k} \zeta e^{i\theta} + L \right\}, L = z_0^2 w''(z_0).$$

Using (12), we have

$$Re \left\{ \frac{z_0 w''(z_0)}{w'(z_0)} \right\} = Re \left\{ \frac{z_0 w''(z_0)}{\zeta e^{i\theta}} \right\} \geq \zeta - 1,$$

that is $Re(e^{-i\theta} L) \geq \zeta(\zeta - 1)$.Since $h(r,s,t) \in H$, we have

$$\begin{aligned} & \left| h\left(z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z_0), z^p \mathcal{H}^{\gamma+1,k}_{p,\alpha,\beta} f(z_0), z^p \mathcal{H}^{\gamma+2,k}_{p,\alpha,\beta} f(z_0) \right) \right| \\ = & \left| h\left(e^{i\theta}, e^{i\theta} + \frac{k}{\gamma} \zeta e^{i\theta}, \frac{k^2}{\gamma(\gamma+1)} \left\{ \frac{\gamma(\gamma+1)}{k^2} e^{i\theta} + \frac{k+2\gamma+1}{k} \zeta e^{i\theta} + L \right\} \right) \right| \geq 1. \end{aligned}$$

This contradicts (14). Therefor, we conclude that $\left| z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z) \right| < 1$.

Using (6) instead of (5), we can prove the following

Theorem 2. *Let $h(r,s,t) \in K$ and $f(z) \in \Sigma_p$ satisfy:*

$$\left(z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z), z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta-1} f(z), z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta-2} f(z) \right) \in D \subset \mathbb{C}^3 \qquad (15)$$

and

$$\left| h\left(z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z), z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta-1} f(z), z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta-2} f(z) \right) \right| < 1. \qquad (16)$$

Then

$$\left| z^p \mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z) \right| < 1.$$

Theorem 3. *Let $h(r,s,t) \in M$ and $f(z) \in \Sigma_p$ satisfy:*

$$\left(\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)} \right) \in D \subset \mathbb{C}^3 \tag{17}$$

and

$$\left| h\left(\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}, \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)} \right) \right| < 1. \tag{18}$$

Then

$$\left| \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)} \right| < 1.$$

Proof. For $f \in \sum_p$, let

$$w(z) = \frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z)},$$

then $w(z)$ is either analytic or meromorphic in $\mathbb{U}$, $w(0) = 1$ and $w(z) \neq 1$. Using (5) we have

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z)} = \frac{1}{\gamma+1} \left\{ \gamma w(z) + k\frac{zw'(z)}{w(z)} + 1 \right\}$$

and

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z)} = \frac{1}{(\gamma+2)} \left\{ \gamma w(z) + k\frac{zw'(z)}{w(z)} + 2 + k^2 \frac{\frac{\gamma}{k} zw'(z) + \frac{zw'(z)}{w(z)} + z^2\frac{w''(z)}{w(z)} - (\frac{zw'(z)}{w(z)})^2}{\gamma w(z) + k\frac{zw'(z)}{w(z)} + 1} \right\}.$$

Claim that $|w(z)| < 1, z \in \mathbb{U}$. If it is not, then there exists a point $z_0 \in \mathbb{U}$, $\underset{|z|<|z_0|}{max} |w(z)| = |w(z)| = 1$. Taking $w(z_0) = e^{i\theta}$ and by Lemma 1 with $a = \tau = 1$, we see that

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma,k} f(z_0)} = w(z_0) = e^{i\theta},$$

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+1,k} f(z_0)} = \frac{1}{\gamma+1} \left(\gamma e^{i\theta} + k\zeta + 1 \right)$$

and

$$\frac{\mathcal{H}_{p,\alpha,\beta}^{\gamma+3,k} f(z_0)}{\mathcal{H}_{p,\alpha,\beta}^{\gamma+2,k} f(z_0)} = \frac{1}{\gamma+2} \left\{ \gamma e^{i\theta} + k\zeta + 2 + k^2 \frac{\frac{\gamma}{k} e^{i\theta} + \zeta - \zeta^2 + L}{\gamma e^{i\theta} + k\zeta + 1} \right\}, L = z_0^2 \frac{w''(z_0)}{w(z_0)}, \zeta > 1.$$

Using (12), we have

$$\Re \left\{ \frac{z_0 w''(z_0)}{w'(z_0)} \right\} = Re \left\{ \frac{z_0 w''(z_0)}{\zeta e^{i\theta}} \right\} \geq \zeta - 1,$$

that is $\Re L \geq \zeta(\zeta-1)$.Since $h(r,s,t) \in M$, we have

$$\left| h\left(\frac{\mathcal{H}^{\gamma+1,k}_{p,\alpha,\beta} f(z_0)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z_0)}, \frac{\mathcal{H}^{\gamma+2,k}_{p,\alpha,\beta} f(z_0)}{\mathcal{H}^{\gamma+1,k}_{p,\alpha,\beta} f(z_0)}, \frac{\mathcal{H}^{\gamma+3,k}_{p,\alpha,\beta} f(z_0)}{\mathcal{H}^{\gamma+2,k}_{p,\alpha,\beta} f(z_0)} \right) \right|$$

$$= \left| h\left(e^{i\theta}, \frac{\gamma e^{i\theta}+k\zeta+1}{\gamma+1}, \frac{1}{(\gamma+2)} \left\{ \gamma e^{i\theta} + k\zeta + 2 + k^2 \frac{\frac{\gamma}{k} e^{i\theta} + \zeta - \zeta^2 + L}{\gamma e^{i\theta} + k\zeta + 1} \right\} \right) \right| \geq 1.$$

This contradicts (18). Therefor, we conclude that $\left| \frac{\mathcal{H}^{\gamma+1,k}_{p,\alpha,\beta} f(z)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z)} \right| < 1$.

Using (6) instead of (5), we can prove the following

Theorem 4. *Let $h(r,s,t) \in N$ and $f(z) \in \Sigma_p$ satisfy:*

$$\left(\frac{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta+1} f(z)}, \frac{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta-1} f(z)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z)}, \frac{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta-2} f(z)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta-1} f(z)} \right) \in D \subset \mathbb{C}^3 \tag{19}$$

and

$$\left| h\left(\frac{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta+1} f(z)}, \frac{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta-1} f(z)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z)}, \frac{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta-2} f(z)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta-1} f(z)} \right) \right| < 1. \tag{20}$$

Then

$$\left| \frac{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta} f(z)}{\mathcal{H}^{\gamma,k}_{p,\alpha,\beta+1} f(z)} \right| < 1.$$

Remark.Putting $p=1$ in the above results, we obtain the results in [1].

3 Open Problem

The authors suggest to study these classes defined by the Frasin [2] operator:

$$\mathcal{L}^m_{\lambda,\mu} f(z) = \frac{1}{z^p} + \sum_{n=1}^{\infty} \left(\frac{1}{1+\lambda(n-p-1)} \right)^m \frac{(\mu)_n}{(1)_n} a_{n-p} z^{n-p}.$$

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