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Application of Interval Type 2 Fuzzy SAW in Flood Control Project

Lazim Abdullah[1], Nurnadiah Zamri[2] and Chun Min Goh[3]

[1,3] School of Informatics and Applied Mathematics,
Universiti Malaysia Terengganu
Malaysia
e-mail: lazim_m@umt.edu.my, chunmingoh@gmail.com

[2] Faculty of Informatics and Computing
Universiti Sultan Zainal Abidin
Malaysia
email: nadiahzamri@unisza.edu.my

Abstract

The fuzzy simple additive weighting (fuzzy SAW) has been applied to resolve numerous multi-criteria decision-making (MCDM) problems, in which triangular fuzzy numbers are employed in describing experts' linguistic evaluation. The recent discovery of interval type-2 fuzzy SAW (IT2FSAW) can offer a new method in solving MCDM problems where interval type-2 fuzzy numbers (IT2FN) are used to define experts' linguistic evaluation. Contrarily to the fuzzy SAW, which directly utilizes triangular fuzzy numbers, this method presents IT2FN to improve evaluation in solving MCDM problems. In this paper, MCDM problem in flood management is investigated where best alternative in flood control project is proposed using the IT2FSAW. Seven alternatives and seven criteria of flood management are identified to construct an MCDM problem. Four experts in flood management were invited to provide linguistic evaluation of alternatives with respect to criteria. The IT2FN linguistic terms were computed using the seven-step procedures of IT2FSAW. Computational output displays that the alternative 'catchment area' is nominated as the best alternative of flood control project. The findings of this study suggest that the authority could provide more catchment areas from which rainfall would flows to a low point.

Keywords: *Simple additive weighting, Interval typ-2 fuzzy number, Linguistic evaluation, Decision making, Flood management.*

1 Introduction

In recent years, multi-criteria decision making (MCDM) methods have been an increasing interest in solving multi-criteria problems. There are hundreds of MCDM methods available in literature and the Simple Additive Weighting (SAW) is one of the many MCDM methods. However, criteria and alternatives of MCDM problems are sometimes very vague and uncertain. Therefore, similar with other MCDM methods, the SAW also has been extended to fuzzy SAW as to deal with subjective evaluation. The fuzzy SAW takes the advantage of linguistic terms in which these linguistic terms can be represented by fuzzy numbers. These representations would allow MCDM problems to be solved systematically. Abdullah & Kamal [1] provided a review of recent efforts in the applications of SAW and fuzzy SAW in solving MCDM methods. In the fuzzy SAW, most of the linguistic terms used are represented by triangular fuzzy numbers where its memberships are given in single value. However, these single value memberships are not good enough to represent vague and uncertain information. Therefore, Mendel [2] proposed interval type-2 fuzzy numbers (IT2FN) where the numbers are given in interval. IT2FN with interval membership grades is germane for dealing with uncertainties and inaccuracy in many real world problems and situation. According to Ceberio & Kreinovich [3] the uncertainty about different perceptions and concepts are one of the examples for IT2FN enable to model linguistic uncertainty. Along this line, the fuzzy SAW also has been developed to interval type-2 fuzzy SAW (IT2FSAW). This method was suggested by Abdullah & Kamal [4], [5] and applied to numerical examples. The results and validation techniques presented in these works has shown the superiority of this method contrast to other MCDM methods. One of the distinctive properties of IT2FSAW is its ability in producing a responsive signed distance based on defuzzification method for solving MCDM problems with linguistic ratings and inadequate preference information.

Despite its advantages in solving incomplete and vague information, applications of the IT2 FSAW method to real case experiments are quite limited. Perhaps it was due to the complexity in handling computations as it involves two layers of membership functions of IT2FN. Of late, there are handfuls of research applying IT2FSAW in solving MCDM problems. For example, Chen [6] indirectly used IT2 fuzzy sets in developing collaborative signed distance-based SAW by defining incomplete information. This method was successfully applied in a case of a cooperative decision-making problem of patient-centered care. Very recently, Abdullah & Kamal [7] applied the IT2FSAW method to a case of ambulance location preference. Since its inception, the applications of IT2FSAW are not fully explored despite the advantages of qualitative linguistic evaluation that represented by IT2FN. In this paper, we extend the application of IT2FSAW to solving flood management problem. According to DeA Brito & Evers [8], flood reduction measures need to be accounted, in order to mitigate the impacts causing by flood. By relating the MCDM methods to flood management, the process tends to be rather uncertain and complex. This is because it would involve multiple stockholders with different backgrounds, knowledge, views, and interests. Thus, the use of decision making tools is required for flood control project where multiple stakeholders, feasible alternatives, views and criteria are considered. The use of MCDM tools, particularly the IT2FSAW can benefit the flood management. This method provides targeted decisions because they are able to handle the inherent uncertainty and complexity problems. Therefore, this paper aims to select the best alternative of flood control project in flood management. This paper contributes to the use of linguistic terms, which allows incorporating IVIFN combines with the SAW in solving flood control projects.

2 Theoretical Background

In this section, the definitions of Interval Type-2 fuzzy sets (IT2FS) and the related definitions of fuzzy SAW are presented.

2.1 Interval type-2 fuzzy sets

IT2FS may be defined from two perspectives. The first definition is based on the major membership and minor membership, and also foot print of uncertainty (FOU). It was mathematically defined by Mendel et al. [9]. For the first definition, a type-2 fuzzy set, $\tilde{A}$ is designated as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$. At this point, $\mu_{\tilde{A}}(x)$ is a type-2 membership function which is actually the composition of an infinite type-1 fuzzy sets. The major fuzzy sets J_x is weighted by minor fuzzy sets $f_x(u)$. In different way, the set can be written as $\tilde{A} = \{((x,u), J_x, f_x(u)) | x \in X; u \in [0,1]\}$, and then could become the solid representation of $\tilde{A}$, The main variable, J_x is the major membership function related to x, u is the minor variable, and $\int_{u \in J_x} f_x(u)/u$ is the minor membership function. Ambiguity about $\tilde{A}$ is projected by the unification of all the major memberships that is known as the Footprint of Uncertainty of $\tilde{A}[FOU(\tilde{A})]$, $FOU(\tilde{A}) = \bigcup_{x \in X} J_x$. Consequently, the FOU develops all the rooted J_x where the minor membership function $f_x(u)/u$ is presented like a weight. These type-2 fuzzy sets are represented as generalized type-2 fuzzy sets, (T2FS), because $f_x(u)/u$ is a type-1 membership function. Interval T2FS, can also be described as an interpretation of T2FS in the aspect that the minor membership function is supposed to be 1.

For the second definition, IT2FS is an extension of T2FS, where the concept of intervals is boldly introduced [10]. An IT2FS $\tilde{A}$ can be described as

$\tilde{A}=\int_{x\in X}\int_{u\in J_x} 1/(x,u)=\int_{x\in X}\left[\int_{u\in J_x} 1/u\right]\Big/x$. While for T2FS, it uses all kind of Type-1 membership functions. An IT2FS differs to a T2FS, because it uses $f_x(u)/u=1$ as a unique weight for each J_x that is being an interval fuzzy set. The FOU of an IT2FS can be restricted by two membership functions which are Upper membership function (UMF) $\overline{\mu}_{\tilde{A}}(x)$ and a Lower membership function (LMF) $\underline{\mu}_{\tilde{A}}(x)$. Interval Type-2 fuzzy sets (IT2FS), $\tilde{A}$ can be described as $\tilde{A}=\int_{x\in X}\int_{u\in J_x} 1/(x,u)=\int_{x\in X}\left[\int_{u\in J_x} 1/u\right]\Big/x$, where $f_x(u)/u=1$ is a distinctive load for each J_x, which coincidently is an interval fuzzy set.

2.2 Fuzzy Simple Additive Weighting

The FSAW is an extension of SAW in which fuzzy numbers are employed instead of crisp numbers. It is proposed as to respond to the fact that the SAW is not always applicable owing to limitations in computational consistency and applications. The FSAW was proposed by Chou et al.[11], where trapezoidal fuzzy numbers are employed to show any fuzziness in scores and weights. The computational procedures of FSAW encompass several fuzzy arithmetic operations [12]. The FSAW method applies fuzzy weighting to approach individual decision makers after taking into account the professional experience and the importance of each decision maker. The concept of FSAW is linked to factor rating system that is based on the fuzzy set theory and the SAW to improve its ability in solving MCDM problems with imprecise criteria. The advantages from these links, can be seen in terms of practical operating mechanism, problem solving measures in the management cycle of plan followed by do, then check and finally action. This management cycle could easily be used in practical applications of any organization [13]. The basic algorithm of FSAW method is summarized as follows.

Start
Step 1: Define Linguistic Variables
Step 2: Introduce linguistic weighting of criteria.
Step 3: Find aggregated fuzzy weights of criteria.
Step 4: Defuzzify the fuzzy weights of criteria.
Step 5: Calculate the normalized weights and create the weight vector.
Step 6: Create ranking order of criteria weights.
End

With some modifications to the basic FSAW, Modarres & Sadi-Nezhad [14] offered a new FSAW method for multi-attribute decision making problems.

Ranking of fuzzy numbers were obtained before utilizing any fuzzy arithmetic in FSAW method. This step is used as to avoid round off errors in defuzzification that probably caused by arithmetic operations such as multiplication or division. They implemented preference ratio concept for the ranking phase, where pairwise comparisons of fuzzy numbers are made. This is just an example of the development of FSAW where fuzzy numbers are vehemently used. As a replacement for using ordinary fuzzy numbers, this paper applies IT2FS combined with SAW in unravelling flood management decision problem.

3 Methodology

In this section, the case related to multi-criteria of flood management problem and the steps used in computation are described as follows.

3.1 Criteria

The criteria that considered in this paper are economic factors (C_1), social factors (C_2) environment factors (C_3), technical factors (C_4) political factors (C_5), legislative factors (C_6) and management factors (C_7) while the alternatives are dam or reservoir (A_1) , dikes (A_2), pumping station (A_3), flood barrier or flood gate (A_4), river basins (A_5) retention pond (A_6), and catchment areas (A_7). All these information are retrieved from the official website of Drainage and Irrigation Department (DID) of Malaysia. Economic factors such as project cost, reliability economic factors, operational cost, and maintenance cost, social factors such as effect on demographic, and social acceptability, environmental factors such as sanitary condition, and rainfall factors, technical factors such as adaptability and technical complexity, management factors such as area used and land used are the criterions that would need to consider in this research. These alternatives and criteria are the main subject of investigation where a group of expert are invited to provide responses on the importance of alternatives in the midst of multiple criteria.

3.2 Experts and data collections

In this research, linguistic data are obtained from a group of expert team which was targeted to assist in linguistic data collections. Personal communication with three DID officers and an academician were conducted. These data provide useful input information for calculating the weights of criteria. Table 1 presents the linguistic terms and IT2FS used.

Table 1: Linguistic terms and IT2FS

Linguistic terms		IT2FS
Very Low	(VL)	((0,0,0,0;1,1),(0,0,0,0.05;0.9,0.9))
Low	(L)	((0.0,0.1,0.1,0.3,0.5;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))
Medium Low	(ML)	((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))
Medium	(M)	((0.3,0.5,0.5,0.7;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))
Medium High	(MH)	((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))
High	(H)	((0.7,0.9,0.9,1;1,1),(0.8,0.9,0.9,0.95;0.9,0.9))
Very High	(VH)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

Source: (Chen & Lee, [15])

The linguistic data and its respective IT2FSs are analysed using the IT2FSAW method.

3.3 Algorithm of IT2FSAW

The IT2FSAW method is presented to conclude the compromise solution for fuzzy MCDM problems. It is proposed that decision makers provide judgment over the rankings of the alternatives with respect to the criterion by using the linguistic expression. A simplified systematic approach of IT2FSAW method retrieved from [7] for multiple criterion decision making is given as follows.

Step 1: Create decision matrix for each decision maker using
$$Y_p = \left(\tilde{\tilde{f}}{}_{ij}^{p}\right)_{m\times n} = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_m \end{array}\begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{f}}{}_{11}^{p} & \tilde{\tilde{f}}{}_{12}^{p} & \cdots & \tilde{\tilde{f}}{}_{1n}^{p} \\ \tilde{\tilde{f}}{}_{21}^{p} & \tilde{\tilde{f}}{}_{22}^{p} & \cdots & \tilde{\tilde{f}}{}_{2n}^{p} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}{}_{m1}^{p} & \tilde{\tilde{f}}{}_{m2}^{p} & \cdots & \tilde{\tilde{f}}{}_{mn}^{p} \end{bmatrix} \end{array}$$
, and construct the aggregated fuzzy rating matrix using $\overline{Y} = \left(\tilde{\tilde{f}}_{ij}\right)_{m\times n}$.

Step 2: Create the weighted matrix of the criterion based on
$$W_p = \left(\tilde{\tilde{w}}{}_{i}^{p}\right)_{1\times m} = \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{bmatrix} \tilde{\tilde{w}}{}_{1}^{p} & \tilde{\tilde{w}}{}_{2}^{p} & \cdots & \tilde{\tilde{w}}{}_{m}^{p} \end{bmatrix} \end{array}$$
and calculate aggregated fuzzy weight, $\overline{W}$ using $\overline{W} = \left(\tilde{\tilde{w}}_{i}\right)_{1\times m}$.

Step 3: Reduce type-2 fuzzy set into type-1 fuzzy set to simplify the equation using $y_l = \dfrac{\sum_{i=1}^{M} f_l^i y_l^i}{\sum_{i=1}^{M} f_l^i}$ and $y_r = \dfrac{\sum_{i=1}^{M} f_r^i y_r^i}{\sum_{i=1}^{M} f_r^i}$.

Defuzzify an interval set from type reduction by using $y_c(x)=\dfrac{\sum_{i=1}^{N} y_i \mu_A(y_i)}{\sum_{i=1}^{N} \mu_A(y_i)}$.

Step 4: Normalize the weight for criterion using $\tilde{\tilde{W}}_j = \dfrac{Y_j}{\sum_{j=1}^{n} Y_j}$, $j = 1,2,\ldots,n$ and create the weight vector.

Step 5: Weighted decision matrix is constructed using the following multiplication,

$$D=\bar{Y}\otimes W=\begin{bmatrix} \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \cdots & \tilde{\tilde{f}}_{1n} \\ \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \cdots & \tilde{\tilde{f}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{m1} & \tilde{\tilde{f}}_{m2} & \cdots & \tilde{\tilde{f}}_{mn} \end{bmatrix} \otimes \begin{bmatrix} \tilde{\tilde{W}}_1 \\ \tilde{\tilde{W}}_2 \\ . \\ . \\ . \\ \tilde{\tilde{W}}_n \end{bmatrix}$$

$$=\begin{bmatrix} \tilde{\tilde{f}}_{11}\otimes\tilde{\tilde{W}}_1 \oplus \tilde{\tilde{f}}_{12}\otimes\tilde{\tilde{W}}_2 \oplus \ldots \tilde{\tilde{f}}_{1n}\otimes\tilde{\tilde{W}}_n \\ \tilde{\tilde{f}}_{12}\otimes\tilde{\tilde{W}}_1 \oplus \tilde{\tilde{f}}_{22}\otimes\tilde{\tilde{W}}_2 \oplus \ldots \tilde{\tilde{f}}_{2n}\otimes\tilde{\tilde{W}}_n \\ . \\ . \\ . \\ \tilde{\tilde{f}}_{m1}\otimes\tilde{\tilde{W}}_1 \oplus \tilde{\tilde{f}}_{m2}\otimes\tilde{\tilde{W}}_2 \oplus \ldots \tilde{\tilde{f}}_{mn}\otimes\tilde{\tilde{W}}_n \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{A}}_1 \\ \tilde{\tilde{A}}_2 \\ . \\ . \\ . \\ \tilde{\tilde{A}}_i \end{bmatrix}$$

Step 6: Calculate the ranking value of IT2 FS.

First construct the upper fuzzy preference matrix P^U, where the element of this matrix is the possibility of single membership compared to other single memberships. The general matrix of upper fuzzy preference can be seen in [15]. The possibility $p\left(\tilde{A}_1^U \geq \tilde{A}_1^U\right)$ can be computed as follows:

$$p\left(\tilde{A}_s^U \geq \tilde{A}_t^U\right) = \max\left(1-\max\left(E_{ts},0\right),0\right) = \max\left(1-\max\left(\frac{N_{ts}}{D_{ts}},0\right),0\right) \quad (1)$$

$$=\max\left(1-\max\left(\frac{\sum_{k=1}^{4}\max\left(a_{tk}^U - a_{sk}^U,0\right)+\left(a_{t4}^U - a_{s1}^U,0\right)+\sum_{k=1}^{2}\max\left(\begin{matrix} H_k\left(\tilde{A}_t^U\right) \\ -H_k\left(\tilde{A}_s^U\right),0 \end{matrix}\right)}{\sum_{k=1}^{4}\left|a_{tk}^U - a_{sk}^U\right|+\left(a_{s4}^U - a_{s1}^U\right)+\left(a_{t4}^U - a_{t1}^U\right)+\sum_{k=1}^{2}\left|\begin{matrix} H_k\left(\tilde{A}_t^U\right) \\ -H_k\left(\tilde{A}_s^U\right) \end{matrix}\right|},0\right),0\right) \quad (2)$$

Similarly, the lower fuzzy preference matrix P^L can be constructed. The full general matrix of lower preference can be retrieved from [15].

Therefore, $$Rank\left(\tilde{A}_i^U\right)=\frac{1}{n(n-1)}\left(\sum_{k=1}^{n} p\left(\tilde{A}_i^U \geq \tilde{A}_k^U\right)+\frac{n}{2}-1\right) \quad (3)$$

and $$Rank\left(\tilde{A}_i^L\right)=\frac{1}{n(n-1)}\left(\sum_{k=1}^{n} p\left(\tilde{A}_i^L \geq \tilde{A}_k^l\right)+\frac{n}{2}-1\right) \quad (4)$$

The final ranking values can be obtained by adding upper preference matrix and lower preference mtrix, then divided it by 2. It is an average operation of two preference matrices.

Step 7: Rank the alternatives.
The seven-step of the IT2FSAW is ended with a preference or ranking of alternatives.

4 Implementation and Results

In this paper, IT2FSAW is used to calculate the weights of each criterion and then ranked the alternatives accordingly.

The algorithm of IT2FSAW (see Section 3.3) is implemented as follows.

Step 1: The aggregated fuzzy rating matrix is created from the decision matrices obtained from decision makers.

$$\bar{Y} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 \end{array} \\ \begin{bmatrix} \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \tilde{\tilde{f}}_{13} & \tilde{\tilde{f}}_{14} & \tilde{\tilde{f}}_{15} & \tilde{\tilde{f}}_{16} & \tilde{\tilde{f}}_{17} \\ \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \tilde{\tilde{f}}_{23} & \tilde{\tilde{f}}_{24} & \tilde{\tilde{f}}_{25} & \tilde{\tilde{f}}_{26} & \tilde{\tilde{f}}_{27} \\ \tilde{\tilde{f}}_{31} & \tilde{\tilde{f}}_{32} & \tilde{\tilde{f}}_{33} & \tilde{\tilde{f}}_{34} & \tilde{\tilde{f}}_{35} & \tilde{\tilde{f}}_{36} & \tilde{\tilde{f}}_{37} \\ \tilde{\tilde{f}}_{41} & \tilde{\tilde{f}}_{42} & \tilde{\tilde{f}}_{43} & \tilde{\tilde{f}}_{44} & \tilde{\tilde{f}}_{45} & \tilde{\tilde{f}}_{46} & \tilde{\tilde{f}}_{47} \\ \tilde{\tilde{f}}_{51} & \tilde{\tilde{f}}_{52} & \tilde{\tilde{f}}_{53} & \tilde{\tilde{f}}_{54} & \tilde{\tilde{f}}_{55} & \tilde{\tilde{f}}_{56} & \tilde{\tilde{f}}_{57} \\ \tilde{\tilde{f}}_{61} & \tilde{\tilde{f}}_{62} & \tilde{\tilde{f}}_{63} & \tilde{\tilde{f}}_{64} & \tilde{\tilde{f}}_{65} & \tilde{\tilde{f}}_{66} & \tilde{\tilde{f}}_{67} \\ \tilde{\tilde{f}}_{71} & \tilde{\tilde{f}}_{72} & \tilde{\tilde{f}}_{73} & \tilde{\tilde{f}}_{74} & \tilde{\tilde{f}}_{75} & \tilde{\tilde{f}}_{76} & \tilde{\tilde{f}}_{77} \end{bmatrix} \end{array} \quad (5)$$

where

$$f_{11}=\frac{\text{MH+M+M+MH}}{4}$$
$$=((0.4,0.6,0.6,0.8;1,1),(0.5,0.6,0.6,0.7;0.9,0.9))$$

$f_{12} = ((0.5,0.675,0.675,0.825;1,1),(0.5875,0.675,0.675,0.75;0.9,0.9))$
$f_{13} = ((0.5,0.675,0.675,0.825;1,1),(0.5875,0.675,0.675,0.75;0.9,0.9))$
$f_{14} = ((0.65,0.825,0.825,0.95;1,1),(0.7375,0.825,0.825,0.8875;0.9,0.9))$
$f_{15} = ((0.5,0.7,0.7,0.85;1,1),(0.6,0.7,0.7,0.775;0.9,0.9))$

All other elements in the matrix are computed similarly.

Step 2: The weighting matrix of the criterion is constructed and the aggregated fuzzy weight is constructed.

$$\overline{W} = \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \\ \end{matrix}\left[\begin{matrix} \approx \atop w_1 & \approx \atop w_2 & \approx \atop w_3 & \approx \atop w_4 & \approx \atop w_5 & \approx \atop w_6 & \approx \atop w_7 \end{matrix}\right] \quad (6)$$

where

$$w_1 = \frac{\text{VH} + \text{M} + \text{VH} + \text{M}}{4}$$

$= ((0.6,0.75,0.75,0.85;1,1),(0.675,0.75,0.75,0.8;0.9,0.9))$
$w_2 = ((0.75,0.9,0.9,0.975;1,1),(0.825,0.9,0.9,0.9375;0.9,0.9))$
$w_3 = ((0.8,0.95,0.95,1;1,1),(0.875,0.95,0.95,0.975;0.9,0.9))$
$w_4 = ((0.55,0.75,0.75,0.9;1,1),(0.65,0.75,0.75,0.825;0.9,0.9))$
$w_5 = ((0.5,0.7,0.7,0.875;1,1),(0.6,0.7,0.7,0.7875;0.9,0.9))$
$w_6 = ((0.4,0.6,0.6,0.8;1,1),(0.5,0.6,0.6,0.7;0.9,0.9))$
$w_7 = ((0.4,0.6,0.6,0.775;1,1),(0.5,0.6,0.6,0.6875;0.9,0.9))$

Step 3: Type-2 fuzzy set is condensed into type-1 fuzzy set and then the interval set is defuzzified from type reduction.

$$y_l^1 = \left\{\frac{0.6 + 0.75 + 0.75 + 0.85}{2}\right\} = \left\{\frac{2.95}{2}\right\} = \{1.475\}$$

$y_l^2 = \{1.7625\}$, $y_l^3 = \{1.85\}$, $y_l^4 = \{1.475\}$, $y_l^5 = \{1.3875\}$
$y_l^6 = \{1.2000\}$, $y_l^7 = \{1.1875\}$
$y_l = \{1.475,1.7625,1.85,1.475,1.3875,1.2,1.1875.\}$

$$y_r^1 = \left\{\frac{0.675 + 0.75 + 0.75 + 0.8}{2}\right\} = \left\{\frac{2.975}{2}\right\} = \{1.4875\}$$

$y_r^2 = \{1.7813\}$, $y_r^3 = \{1.875\}$
$y_r^4 = \{1.4875\}$, $y_r^5 = \{1.3937\}$
$y_r^6 = \{1.2000\}$, $y_r^7 = \{1.1938\}$
$y_l = \{1.4875,1.7813,1.875,1.4875,1.3937,1.2,1.1938.\}$

$$y_c^1(x) = \left\{\frac{1.475 + 1.4875}{2}\right\} = \left\{\frac{2.9625}{2}\right\} = \{1.4813\}$$

$y_c^2(x) = \{1.7719\}$, $y_c^3(x) = \{1.8625\}$

$y_c^4(x) = \{1.4812\}$, $y_c^5(x) = \{1.3906\}$

$y_c^6(x) = \{1.2000\}$, $y_c^7(x) = \{1.1906\}$

$Y = \{1.4813, 1.7719, 1.8625, 1.4812, 1.3906, 1.2000, 1.1906\}$

Step 4: The weight for criterion is normalized and the weight vector is constructed.

$\tilde{\tilde{W}}_1 = (0.1427)$ $\tilde{\tilde{W}}_2 = (0.1707)$, $\tilde{\tilde{W}}_3 = (0.1795)$, $\tilde{\tilde{W}}_4 = (0.1427)$

$\tilde{\tilde{W}}_5 = (0.1340)$, $\tilde{\tilde{W}}_6 = (0.1156)$, $\tilde{\tilde{W}}_7 = (0.1147)$

$\overline{W} = [0.1427 \quad 0.1707 \quad 0.1795 \quad 0.1427 \quad 0.1340 \quad 0.1156 \quad 0.1147]$

Step 5: The weighted decision matrix is created as follow.

$\tilde{\tilde{d}}_1 =$

$(0.4573, 0.6530, 0.6530, 0.8255; 1, 1), (0.5552, 0.6530, 0.6530, 0.7393; 0.9, 0.9)$

$\tilde{\tilde{d}}_2 =, (0.3682, 0.5341, 0.5341, 0.6955; 1, 1), (0.4512, 0.5341, 0.5341, 0.6194; 0.9, 0.9)$

$\tilde{\tilde{d}}_3 =, (0.4649, 0.6351, 0.6351, 0.7813; 1, 1), (0.55, 0.6351, 0.6351, 0.7127; 0.9, 0.9)$

$\tilde{\tilde{d}}_4 =, (0.4663, 0.6507, 0.6507, 0.8058; 1, 1), (0.5585, 0.6507, 0.6507, 0.7297; 0.9, 0.9)$

$\tilde{\tilde{d}}_5 =, (0.4251, 0.5984, 0.5984, 0.7489; 1, 1), (0.5118, 0.5984, 0.5984, 0.6765; 0.9, 0.9)$

$\tilde{\tilde{d}}_6 =, (0.4509, 0.6204, 0.6204, 0.774; 1, 1), (0.5357, 0.6204, 0.6204, 0.7017; 0.9, 0.9)$

$\tilde{\tilde{d}}_7 =, (0.5537, 0.7093, 0.7093, 0.8236; 1, 1), (0.6315, 0.7093, 0.7093, 0.7710; 0.9, 0.9)$

Step 6: Ranking of the value of IT2FS is calculated.

The likelihood of upper membership can be calculated as follows.

$p(\tilde{A}_1^U \geq \tilde{A}_1^U) = 0.5$, $p(\tilde{A}_1^U \geq \tilde{A}_2^U) = 0.7933$, $p(\tilde{A}_1^U \geq \tilde{A}_3^U) = 0.5707$,
$p(\tilde{A}_1^U \geq \tilde{A}_4^U) = 0.5175$, $p(\tilde{A}_1^U \geq \tilde{A}_5^U) = 0.6796$, $p(\tilde{A}_1^U \geq \tilde{A}_6^U) = 0.6111$,
$p(\tilde{A}_1^U \geq \tilde{A}_7^U) = 0.3224$.

Similar computations are implemented to other upper likelihood memberships.

The likelihood of lower memberships can be calculated as follow.

$p(\tilde{A}_1^L \geq \tilde{A}_2^L) = 0.9211$, $p(\tilde{A}_1^L \geq \tilde{A}_3^L) = 0.6199$, $p(\tilde{A}_1^L \geq \tilde{A}_4^L) = 0.5228$,

$p\left(\tilde{A}_1^L \geq \tilde{A}_5^L\right) = 0.7849$, $p\left(\tilde{A}_1^L \geq \tilde{A}_6^L\right) = 0.6897$, $p\left(\tilde{A}_1^L \geq \tilde{A}_7^L\right) = 0.1980$.
Similar computations are implemented to other lower likelihood memberships.

Then,

$Rank\left(\tilde{A}_1^U\right) = 0.1546$

$Rank\left(\tilde{A}_2^U\right) = 0.1024$, $Rank\left(\tilde{A}_3^U\right) = 0.1452$

$Rank\left(\tilde{A}_4^U\right) = 0.1527$, $Rank\left(\tilde{A}_5^U\right) = 0.1261$

$Rank\left(\tilde{A}_6^U\right) = 0.1387$, $Rank\left(\tilde{A}_7^U\right) = 0.1802$

$Rank\left(\tilde{A}_1^L\right) = 0.1604$

$Rank\left(\tilde{A}_2^L\right) = 0.0848$, $Rank\left(\tilde{A}_3^L\right) = 0.1465$

$Rank\left(\tilde{A}_4^L\right) = 0.1583$, $Rank\left(\tilde{A}_5^L\right) = 0.1168$

$Rank\left(\tilde{A}_6^L\right) = 0.1352$, $Rank\left(\tilde{A}_7^L\right) = 0.1980$

$Rank\left(\tilde{\tilde{A}}_1\right) = 0.1575$

$Rank\left(\tilde{\tilde{A}}_2\right) = 0.0936$, $Rank\left(\tilde{\tilde{A}}_3\right) = 0.1459$, $Rank\left(\tilde{\tilde{A}}_4\right) = 0.1555$

$Rank\left(\tilde{\tilde{A}}_5\right) = 0.1215$, $Rank\left(\tilde{\tilde{A}}_6\right) = 0.1369$, $Rank\left(\tilde{\tilde{A}}_7\right) = 0.1891$

Step 7: The value of $Rank\left(\tilde{\tilde{A}}_i\right)$, characterizes the degree of favor of the alternatives. Therefore,

$$Rank\left(\tilde{\tilde{A}}_7\right) \succ Rank\left(\tilde{\tilde{A}}_1\right) \succ Rank\left(\tilde{\tilde{A}}_4\right) \succ Rank\left(\tilde{\tilde{A}}_3\right) \succ Rank\left(\tilde{\tilde{A}}_6\right) \succ Rank\left(\tilde{\tilde{A}}_5\right) \succ Rank\left(\tilde{\tilde{A}}_2\right)$$

Thus, catchment area (A_7) is selected as the best alternatives in flood management followed by dam or reservoir (A_1). The least preferred alternative is dikes (A_2).

As pointed out in the introduction to this paper, there are seven alternatives and seven criteria of flood mitigation projects that have been analysed. The seven-step computation procedure of IT2FSAW has been implemented and the final result concludes that 'catchment area' with degree of likelihood at 0. 1891 is chosen as the best alternative in flood mitigation project. Contrary to the result obtained from the present study, Taib et al,.[16] found out that 'reservoir' was the best solution in flood mitigation project. However, the result is not somewhat surprising as 'reservoir' is the second best solution in the present study. The degree of likelihood between these two alternatives is not much differed, barely at 0.0316. The difference in final results would be caused by different method used, of which IT2FSs are worked on different definition compared to bi-conflicting fuzzy sets.

The IT2FSAW method was able to capture the likelihood for alternatives of flood control project. The vagueness of criteria and alternatives in flood management can be dealt using the decision method IT2FSAW. It is also good to note that qualitative linguistic evaluations made by decision makers are another point that can explain the results. In this paper, the seven scales of linguistic terms were translated into IVIFN where the vagueness of these linguistic terms are dealt with fuzzy numbers. The decision making method of the current study are consistent with those of researchers [17], [18] who found flood management is indeed a MCDM problem where several qualitative criteria and alternatives could be dealt with the use of various generalisations of fuzzy set theory.

5 Conclusion

MCDM method is a technique that would give effective and useful framework in order to evaluating alternatives from multiple and conflicted criteria. The aim of this research was to decide the best alternative of flood control project in managing flood using the IT2FSAW method. Seven alternatives and seven criteria in flood management have been considered in this paper. The IT2F SAW was successfully selecting the best alternatives of flood control project in flood management based on the weight of criteria that computed from linguistic evaluation given by four experts. Despite the differences in the evaluations among the experts, the method concludes that catchment areas as the best way to mitigate or control floods. This result implicates that the local authority should provide more areas from which rainfall would flows into a river. The result may be useful to the government particularly in solving one of the most environmental disasters in Malaysia. However, the stability of the results is subjected to further investigation. This endeavor could be left for future study.

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