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Targeted Ranking-Based Clustering Using AHP K-Means

Suhailan Safei[1,2], Abdul Samad Shibghatullah[1], MA Burhanuddin[1], and Mohd Kamir Yusof[2]

[1]Faculty of Information and Communication Technology,
Universiti Teknikal Malaysia Melaka, 76100, Melaka, Malaysia
e-mail: samad@utem.edu.my, burhanuddin@utem.edu.my
[2]Faculty of Informatics and Computing,
Universiti Sultan Zainal Abidin, 22200, Terengganu, Malaysia
e-mail: suhailan@unisza.edu.my, mohdkamir@unisza.edu.my

Abstract

K-Means can group similar objects features into specified number (K) of cluster centers region. Similarity is measured based on their closest distance of multiple features coordinate location. However, such distance measurement can be doubtful in satisfying certain clustering application as it does not distinguish the meaning of object features representation. Ordinal feature for example may denote to certain ranking objects rather than just number representation. Thus, clustering result should also consider the existing rank label on these objects instead of distance measurement. New AHP K-Means technique is proposed to preserve rank order for each object in the clustering result. It transforms weighted multi-features objects by aggregating them as a single ranking objects using pair-wise comparing among the objects. These ranking objects are then processed by K-Means based on cluster centers that initially setup on fair distributed ranking scale. Based on experiment using weighted course marks of 92 students, the proposed technique shows that ranking-based clustering using AHP can give accurate ranked clustering result compared to normal weighted K-Means.

Keywords: *K-Means, AHP, Clustering, Ranking.*

1 Introduction

K-Means is one of most popular clustering techniques and widely used in real problem solutions [1]. It will cluster objects into number (K) of cluster center region based on their closest distance of multiple features coordinate location [2], [3]. This technique starts by choosing random points/objects to be used as initial cluster centers. Then it chooses the closest distance of objects to these cluster centers to be grouped together within the cluster center. There are many techniques to calculate the distance of multi-features object, but Euclidean distance is among the best [4]. Once all of the objects have being assigned into these initial cluster center, new cluster center on each cluster will be recalculated based on points location average (i.e. mean) of assigned objects cluster. Then, objects distance is re-calculated and re-grouped using the new cluster centers. The iteration process in recalculating new cluster centers and reassigning objects will stop when there are no changes to the cluster center in each cluster.

On the other hand, the result of clustering is subjective in the eye of the beholder that requires expert to interpret the result [5]. Literally, by measuring distance on multi-features objects, the meaning of feature representation may not be considered in the clustering process [6]. For example, objects that consist of ordinal features may represent objects with ranking attribute. By knowing such feature label, the clustering result is expected to achieve better accuracy in term of ranking consideration. However, this concern may not be achieved as some objects of the same rank may be possibly grouped into different clusters. This is due to the clustering technique that compares distance of objects to the cluster center, rather than measuring closeness among themselves. Distance measurement is also significantly changed when different features weighting are to be considered. Therefore, it is important to consider meaningful initial cluster centers and handle proper feature weighting for a better clustering result [7].

There are many researches on k-means enhancement that relates to the initial cluster center and feature weighting in getting better clustering result. However, to our knowledge extent, only few of researches that investigate k-means algorithm for targeting better ranking-based clustering. This paper proposes an enhanced technique of k-means by integrating Analytic Hierarchy Process (AHP) to transform weighted multi-features objects into aggregated weighted ranked objects. The technique can improve the initial cluster center to be more meaningful considering ranking representation scale and then achieve better accuracy of ranked-based clustering.

2 Related Works

Initial cluster center to increase accuracy of clustering result have been widely proposed [8]–[14]. Unfortunately, initial cluster center configuration towards targeting cluster with ranking representation is yet to be explored. One of relevant research on initial cluster center is proposed to be based on the minimum and maximum objects at extreme ends of objects distance [15]. However, the proposed technique is more targeting towards separating clusters based on most significant (i.e. furthest) distance of multi-features objects without considering their ranking representation.

Another main issue in clustering is the need to handle feature weighting. It can influence clustering result and needs to be balanced with real data instances to avoid inaccurate result [16]. W-k-Means [17] and Tw-k-Means [18] are among recent researches that extend k-Means algorithm to automatically calculate feature weighting, thus outlier features can be deselected [19] to improve the clustering result. On the other hand, feature weighting is also required to purposely tweak the clustering result. For example, in clustering article based on keywords, there will be a need to differentiate some dominant keywords among other keywords. Weighted-K-Means [20] which includes feature weighting in measuring closest distance has improved accuracy in term of text clustering. Using the algorithm, Euclidean distance formula on each object features (X_{ij}) is needed to be multiplied by its predefined feature weighting (W_j) as in Eq. (1). C_k in the equation refers to cluster centers for each feature-j and D is the total number of features.

Equation 1 *Weighted Euclidean distance.*

$$S_{ik} = \min_{s} \sum_{k=1}^{K} \sqrt{\sum_{j}^{D} W_j * \left(X_{ij} - C_{kj}\right)^2} \tag{1}$$

Looking at ranking consideration, distance measurement will be more distorted when feature weighting is in place. Although clustering is an unsupervised approach, its result can still be tweaked towards targeted cluster characteristic. Targeted or desired cluster [21] is a different approach in clustering technique. It involved with semi-supervised clustering as various real-applications have prior knowledge which may have a form of predefined partial clusters based on features weighting; and features selection. In that case, expert model might be incorporated for targeted data clustering [22].

Currently, no previous study has investigated expert model to tweak clustering result. Many previous studies have only focused on integrating the clustering and ranking separately [23]–[27]. These researches do not investigate clustering effectiveness and potential setup on using ranking objects that initially processed by ranking algorithm. This paper is proposing AHP K-Means algorithm to aggregate multidimensional (i.e. multi-criteria) objects to be used by K-Means in achieving better accuracy of ranking-based clustering.

3 AHP K-Means Algorithm

AHP K-Means algorithm uses AHP ranking algorithm to aggregate multi-features objects into a single ranking objects. The feature of objects must only be in ordinal representation so that the ranking can be meaningful. These aggregated objects will be processed by K-Means using Euclidean distance in clustering them based on closest distance to the cluster centers. Initial setup of cluster centers are donated to the fair distributed of rank points range from the lowest to the highest rank. All the processes involved in this algorithm are depicted in Fig. 1 and the following sub topics are describing the implementation detail.

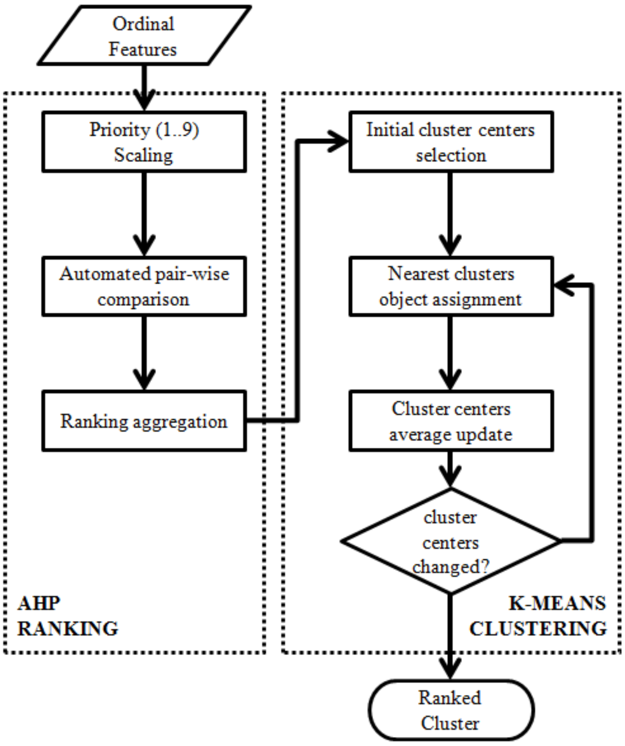

Fig.1: AHP K-Means Algorithm

3.1 Object features scaling

Object features that carry ordinal value need to be transformed into importance level pair-wise scale which is set from 1 to 9 as proposed by Saaty [28]. This process can be automatically done by converting all ordinal objects features (X_{id}) into unity-based normalization and then rescaling them to be in the important level range of 1 to 9 as shown in Eq. (2).

Features priority scaling

$$X'_{id} = \left(\frac{X_{id} - \min_{d} X}{\max_{d} X - \min_{d} X} \right) * 8 + 1 \tag{2}$$

The formula will rate the highest ordinal value as 9 while the lowest as 1. *d* in the equation is referred to the feature that is been scaled. Table 1 shows the sample scaling result calculation on 3 objects (X) that consists of 2 features (d).

Table 1: Important Level Scaling From Ordinal Objects

ID	Carry mark (d=1)	Final mark (d=2)	X'_{i1}	X'_{i2}
1	40	60	1	1
2	50	77	3	6.23
3	80	86	9	9

3.2 Pair-wise objects comparison

Ranking is systematically done in AHP by setting up the priority of an object compared to other objects on each feature. It is done through a pair-wise matrix (A_{ij}) which is formed by having all object-i into its rows and columns. Although the comparison between two objects are supposed to be filled or decided by expert using the importance scale from 1 (equally important) to 9 (absolutely more important), it is possible to automatically represent the level by calculating the ratio of two different objects on the same feature-d. Eq. (3) is used to fill up the diagonal and upper triangular of comparison matrix. *N* in the equation represents the total objects in feature-d.

Automated pair-wise comparison (diagonal and upper triangular)

$$A_{ij-d} = \frac{X'_{i-d}}{X'_{j-d}} \; ; i = 1 \text{ to } N, j = i \text{ to } N \tag{3}$$

Table 2 shows the sample calculation to fill up the diagonal and upper triangular of the matrix on feature-1 which is the rescaled values of carry mark (X'_{i-1}). For each object row, its scaled value is divided with the scaled values in for other object specified in each column. As example, element in row 2 and column 3 for feature-1(A_{23-1}) is filled with X'_{2-1} / X'_{3-1} (3/9).

Table 2: Pair-wise comparison matrix on carry mark feature

i\j	1	2	3
1	1/1	1/3	1/9
2		3/3	3/9
3			9/9

Meanwhile, the lower triangular matrix on each object-i is filled up by using Eq. (4).

Automated pair-wise comparison matrix (lower triangular)

$$A_{ij-d} = \frac{X'_{j-d}}{X'_{i-d}} ; \; i = 2 \text{ to } N, j = 1 \text{ to } (i-1) \tag{4}$$

Table 3 shows the sample calculation to fill up the lower triangular of the matrix. For each object row, its value is the inverse of its scaled value divided by other objects scaled value. As example, element in row 3 and column 2 for feature-1(A_{32-1}) is filled with X'_{3-1} / X'_{2-1} (9/3).

Table 3: Pair-wise comparison matrix on carry mark feature

i\j	1	2	3
1			
2	3/1		
3	9/1	9/3	

This comparison matrix need to be normalized by computing the sum of each column and then divide each column by the corresponding sum as shown in Eq. (5).

Normalized pair-wise matrix

$$A'_{ij-d} = \frac{A_{ij-d}}{\left(\sum_j^N A_{ij-d}\right)} \tag{5}$$

Table 4 shows the sample calculation to normalize the matrix. In each column, calculate the sum of all pair-wise values on each object row. Then for each object row, divide each pair-wise value with its column summation value. As example, total value of column 2 is 4.333 (1/3 + 3/3 + 9/3). Thus, A'_{21} is filled with (1/3)/4.333, A'_{22} is filled with (3/3)/4.333 and A'_{23} is filled with (9/3)/4.333.

Table 4: Normalized pair-wise comparison matrix on carry mark feature

i\j	1	2	3
1	(1/1) / (1+3+9)	(1/3) / (1/3 + 1 + 9/3)	(1/9) / (1/9 + 3/9 + 1)
2	3 / (1+3+9)	(3/3) / (1/3 + 1 + 9/3)	(3/9) / (1/9 + 3/9 + 1)
3	9 / (1+3+9)	(9/3) / (1/3 + 1 + 9/3)	(9/9) / (1/9 + 3/9 + 1)

3.3 Objects ranking aggregation

Each object on each feature is then will be assigned with a new calculated priority point by calculating the average of all its comparison objects using Eq. (6). *W* is the weight for each feature-d. These weights should be configured so they are non-negative and sum to 1.

Priority point

$$P_{id} = W_d * \frac{\sum_j^N A'_{ijd}}{N} \tag{6}$$

Table 5 shows the sample calculation to calculate priority points on each object for carry mark feature. Based on previous value of normalized pair-wise comparison matrix, total up all column values in each object row to get the object priority point of that particular feature. As example, for object row-2, its P_2 is calculated by total up all its column value of A'_{21}, A'_{22} and A'_{23}, and then divide it with the weight of feature-1 (W_1=0.4).

Table 5: Priority points on each object for carry mark feature

i\j	1	2	3	R_{i1}
1	0.077	0.077	0.077	0.077 * 0.4
2	0.231	0.231	0.231	0.231 * 0.4
3	0.692	0.692	0.692	0.692 * 0.4

Finally, in order to aggregate the object multi-features to become a single rank object, the average of priority point from each features for an object need to be calculated using Eq. (7). *D* in this equation refers to the total number of features.

Ranking object

$$R_i = \frac{\sum_d^D P_{id}}{D} \tag{7}$$

3.4 Cluster center initialization

Cluster center can be initially determined based on ranking scale of the aggregated ranking objects (R) by using Eq. (8).

Cluster center selection

$$c_k = \min R + \left[(k-1) * \left(\frac{\max R - \min R}{K-1}\right)\right] \tag{8}$$

In equation 8, K is the total intended cluster number, minR and maxR is the lowest and maximum objects in R_i respectively. Using this initial center cluster distribution, the result of clustering is targeted to converge into local minima based on fair ranking scale distribution. Thus, each cluster can represent certain rank ranging from lowest to the highest cluster.

3.5 Clustering assignment

Each ranking object will be assigned as set of cluster center objects (S_k) if it has the closest distance from the initial cluster center (C) using Euclidean measurement as shown in Eq. (9).

Distance measurement

$$S_{ik} = \min_{s} \sqrt{(R_i - C_k)^2};\, k = 1..K \tag{9}$$

Once all objects have been assigned to certain cluster center, new cluster center need to be recalculated based on current set of assigned objects. New cluster center is calculated by considering the average (i.e. mean) points among the assigned objects on each cluster. The formula is shown in Eq. (10). M in the equation is referred to number of assigned objects in cluster-k.

Cluster center point

$$C_k = \frac{\sum_i^M R_{ik}}{M} \tag{10}$$

Iteratively, this stage is repeated until there is no change on cluster center for all clusters. This means that all objects are successfully assigned to the closest point on each cluster.

4 Result and Discussion

Data set consists of two assessments (coursework and final examination mark) of 92 students on Computer and Organization (COA) subject conducted at Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin in 2013/2014 session. Coursework (CW) and final examination weight was set to 0.6 and 0.4 respectively. Two experiments were performed to compare the clustering result based on rank representation with feature weighting consideration; Weighted K-Means [20] and our proposed AHP-K-Means. A full data set with the clustering result is presented in Table 6. K is set to 3 and initial center clusters are determined based on equation 8 to target the cluster result to be in lowest, average and highest of objects. Fig. 2 shows the result of Weighted K-Means and AHP K-Means.

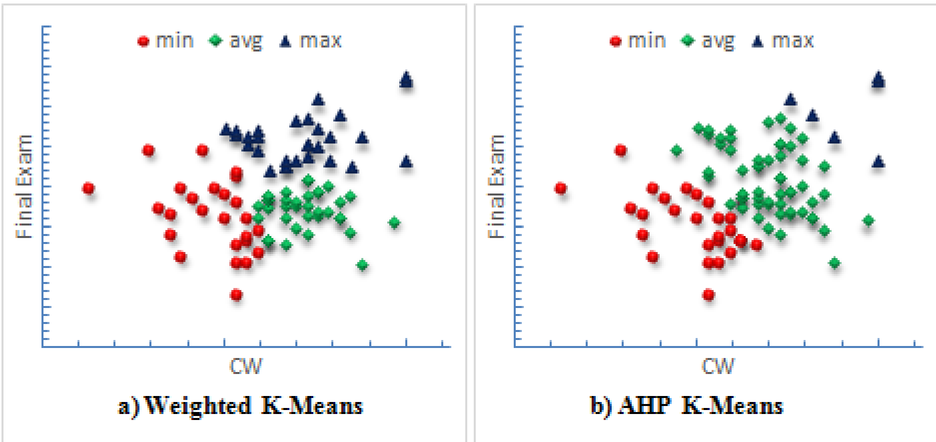

Fig. 1: Ranking-based clustering result using (a) Weighted K-Means and (b) AHP K-Means

From the graph, object ranking representation is moving from lowest to highest rank in diagonal direction. This targeted ranking clustering representation is clearly achieved by AHP K-Means. In other hand, weighted K-Means manage to group closest objects together but did not fully represent the ranking order consideration. The detail observation of clustered objects in Table 7 also proved that AHP K-Means has clustered the objects as the same order of the objects ranking order based on their total marks. However, when using weighted K-Means, many of the objects were clustered without considering the objects ranking order. Normal cluster distribution (i.e. Bell curve) is also most likely achieved in AHP K-Means clustering where the average objects are more significant than the lowest and highest. Some other interesting finding is that the clustering iteration process was also significantly minimized by using AHP K-Means. It just got 2 iterations on AHP K-Means compared to Weighted K-Means that took 7 clustering iterations.

In order to validate the clustering result, purity method is used as in Eq. (11). In this case, N is a total number of objects, t_j is targeted ranking classification, c_k is cluster-k and $max_k j$ is the highest count of ranking classification for each cluster-k.

Purity validation

$$Purity = \frac{1}{N}\sum_{k=1}^{K} \max_{k} j\left|c_k \cap t_j\right| \quad (11)$$

In order to compare the result using purity validation, objects are classified into 3 groups; minimum object (t_1) is set to the total marks of below than 50, average object (t_2) is between 50 and 80, and maximum object (t_3) is above 80. N is 92. Table 6 shows the count number of object ranking classification in each clusters result. The result proves that AHP K-Means has greater purity value (0.9565) compared to Weighted K-Means (0.8913) which represent good representation of ranking-based clustering.

Table 6: Purity validation

	Weighted K-Means				AHP K-Means			
	t_1(50)	t_2(80)	t_3(100)	max-j	t_1(50)	t_2(80)	t_3(100)	max-j
C_1(min)	4	21	0	21	4	23	0	23
C_2(avg)	0	38	0	38	0	59	0	59
C_3(max)	0	23	6	23	0	0	6	6
Total	(21+38+23)			82	(23+59+6)			88
Purity	(82/92)			0.8913	(88/92)			0.9565

5 Conclusion

Clustering based on ranking consideration is a semi unsupervised learning where the object ranking order is known already. Such rating label which also influenced by pre-defined weighting can be used to target the cluster based on their ranking order. However, basic K-Means algorithm put more consideration on closest distance rather than ranking measurement. This paper proposes a new ranking-based clustering algorithm on K-Means by integrating AHP to alter the multi-features objects into meaningful aggregated ranked objects. The validity of the transformed ranked objects to the clustering result accuracy is measured by comparing to the weighted K-Means algorithm. Based on the experiment, it shows that the AHP K-Means method produces better meaningful initial cluster centers, thus improve the accuracy of ranking-based clustering result compared to Weighted K-Means. The iteration to complete the clustering process is also minimized up to 3 times. On the other hand, this research also contributes new method to compare current ranking algorithms accuracy by measuring applicability of their aggregated results using clustering algorithm. Thus, integrating other ranking algorithms into clustering algorithms will be interesting future research to be pondered.

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Table 7: Clustering result using Weighted K-Means and AHP-K-Means

Multi-features Objects						Scaled Objects (1-9)		Aggregated Objects	K-Means Clustering Result	
ID	CW Marks (100)	Final Mark (100)	CW' Marks (0.6)	Final' Mark (04)	Total Marks (100)	CW	Final	AHP Rank Point (R_i)	Weighted K-Means	AHP K-Means
1	42.00	49.00	25.20	19.60	44.80	1	4.882353	0.00548	min	min
2	59.00	28.00	35.40	11.20	46.60	3.344828	2.411765	0.00592	min	min
3	69.00	16.00	41.40	6.40	47.80	4.724138	1	0.00622	min	min
4	57.00	35.00	34.20	14.00	48.20	3.068966	3.235294	0.00635	min	min
5	55.00	43.00	33.00	17.20	50.20	2.793103	4.176471	0.00687	min	min
6	57.00	41.00	34.20	16.40	50.60	3.068966	3.941176	0.00698	min	min
7	69.00	26.00	41.40	10.40	51.80	4.724138	2.176471	0.00727	min	min
8	71.00	26.00	42.60	10.40	53.00	5	2.176471	0.00758	min	min
9	69.00	32.00	41.40	12.80	54.20	4.724138	2.882353	0.0079	min	min
10	63.00	42.00	37.80	16.80	54.60	3.896552	4.058824	0.00801	min	min
11	59.00	49.00	35.40	19.60	55.00	3.344828	4.882353	0.00813	min	min
12	61.00	46.00	36.60	18.40	55.00	3.62069	4.529412	0.00812	min	min
13	73.00	29.00	43.80	11.60	55.40	5.275862	2.529412	0.00821	min	min
14	71.00	33.00	42.60	13.20	55.80	5	3	0.00831	min	min
15	67.00	40.00	40.20	16.00	56.20	4.448276	3.823529	0.00843	min	min
16	53.00	61.00	31.80	24.40	56.20	2.517241	6.294118	0.00845	min	min
17	71.00	34.00	42.60	13.60	56.20	5	3.117647	0.00842	min	min
18	73.00	36.00	43.80	14.40	58.20	5.275862	3.352941	0.00894	min	min
19	75.00	33.00	45.00	13.20	58.20	5.551724	3	0.00894	avg	min
20	75.00	33.00	45.00	13.20	58.20	5.551724	3	0.00894	avg	min
21	75.00	33.00	45.00	13.20	58.20	5.551724	3	0.00894	avg	min
22	65.00	49.00	39.00	19.60	58.60	4.172414	4.882353	0.00906	min	min
23	71.00	40.00	42.60	16.00	58.60	5	3.823529	0.00905	min	min
24	67.00	47.00	40.20	18.80	59.00	4.448276	4.647059	0.00916	min	min
25	69.00	45.00	41.40	18.00	59.40	4.724138	4.411765	0.00926	min	min
26	78.00	32.00	46.80	12.80	59.60	5.965517	2.882353	0.0093	avg	min
27	73.00	40.00	43.80	16.00	59.80	5.275862	3.823529	0.00936	avg	min
28	73.00	44.00	43.80	17.60	61.40	5.275862	4.294118	0.00978	avg	avg
29	63.00	61.00	37.80	24.40	62.20	3.896552	6.294118	0.01001	min	avg
30	75.00	43.00	45.00	17.20	62.20	5.551724	4.176471	0.00999	avg	avg
31	69.00	53.00	41.40	21.20	62.60	4.724138	5.352941	0.0101	min	avg
32	78.00	40.00	46.80	16.00	62.80	5.965517	3.823529	0.01014	avg	avg
33	80.00	37.00	48.00	14.80	62.80	6.241379	3.470588	0.01013	avg	avg
34	69.00	54.00	41.40	21.60	63.00	4.724138	5.470588	0.01021	min	avg
35	82.00	35.00	49.20	14.00	63.20	6.517241	3.235294	0.01024	avg	avg
36	75.00	46.00	45.00	18.40	63.40	5.551724	4.529412	0.0103	avg	avg
37	75.00	46.00	45.00	18.40	63.40	5.551724	4.529412	0.0103	avg	avg
38	75.00	46.00	45.00	18.40	63.40	5.551724	4.529412	0.0103	avg	avg
39	75.00	47.00	45.00	18.80	63.80	5.551724	4.647059	0.01041	avg	avg
40	75.00	47.00	45.00	18.80	63.80	5.551724	4.647059	0.01041	avg	avg
41	78.00	44.00	46.80	17.60	64.40	5.965517	4.294118	0.01056	avg	avg
42	78.00	46.00	46.80	18.40	65.20	5.965517	4.529412	0.01077	avg	avg
43	80.00	44.00	48.00	17.60	65.60	6.241379	4.294118	0.01087	avg	avg
44	82.00	41.00	49.20	16.40	65.60	6.517241	3.941176	0.01086	avg	avg
45	92.00	26.00	55.20	10.40	65.60	7.896552	2.176471	0.01085	avg	avg
46	78.00	48.00	46.80	19.20	66.00	5.965517	4.764706	0.01098	avg	avg
47	80.00	45.00	48.00	18.00	66.00	6.241379	4.411765	0.01097	avg	avg
48	82.00	44.00	49.20	17.60	66.80	6.517241	4.294118	0.01118	avg	avg
49	84.00	41.00	50.40	16.40	66.80	6.793103	3.941176	0.01118	avg	avg
50	75.00	55.00	45.00	22.00	67.00	5.551724	5.588235	0.01124	max	avg
51	84.00	42.00	50.40	16.80	67.20	6.793103	4.058824	0.01128	avg	avg
52	67.00	68.00	40.20	27.20	67.40	4.448276	7.117647	0.01136	max	avg
53	69.00	66.00	41.40	26.40	67.80	4.724138	6.882353	0.01146	max	avg
54	71.00	63.00	42.60	25.20	67.80	5	6.529412	0.01146	max	avg
55	82.00	47.00	49.20	18.80	68.00	6.517241	4.647059	0.01149	avg	avg
56	69.00	67.00	41.40	26.80	68.20	4.724138	7	0.01157	max	avg
57	73.00	61.00	43.80	24.40	68.20	5.275862	6.294118	0.01156	max	avg
58	86.00	42.00	51.60	16.80	68.40	7.068966	4.058824	0.01159	avg	avg

59	90.00	36.00	54.00	14.40	68.40	7.62069	3.352941	0.01158	avg	avg
60	71.00	65.00	42.60	26.00	68.60	5	6.764706	0.01167	max	avg
61	88.00	40.00	52.80	16.00	68.80	7.344828	3.823529	0.01169	avg	avg
62	88.00	40.00	52.80	16.00	68.80	7.344828	3.823529	0.01169	avg	avg
63	78.00	56.00	46.80	22.40	69.20	5.965517	5.705882	0.01182	max	avg
64	78.00	56.00	46.80	22.40	69.20	5.965517	5.705882	0.01182	max	avg
65	84.00	48.00	50.40	19.20	69.60	6.793103	4.764706	0.01191	avg	avg
66	84.00	48.00	50.40	19.20	69.60	6.793103	4.764706	0.01191	avg	avg
67	73.00	65.00	43.80	26.00	69.80	5.275862	6.764706	0.01198	max	avg
68	78.00	58.00	46.80	23.20	70.00	5.965517	5.941176	0.01203	max	avg
69	82.00	52.00	49.20	20.80	70.00	6.517241	5.235294	0.01202	avg	avg
70	82.00	52.00	49.20	20.80	70.00	6.517241	5.235294	0.01202	avg	avg
71	73.00	67.00	43.80	26.80	70.60	5.275862	7	0.01219	max	avg
72	88.00	45.00	52.80	18.00	70.80	7.344828	4.411765	0.01222	avg	avg
73	80.00	58.00	48.00	23.20	71.20	6.241379	5.941176	0.01234	max	avg
74	86.00	50.00	51.60	20.00	71.60	7.068966	5	0.01243	avg	avg
75	82.00	59.00	49.20	23.60	72.80	6.517241	6.058824	0.01275	max	avg
76	90.00	47.00	54.00	18.80	72.80	7.62069	4.647059	0.01274	avg	avg
77	82.00	63.00	49.20	25.20	74.40	6.517241	6.529412	0.01317	max	avg
78	82.00	63.00	49.20	25.20	74.40	6.517241	6.529412	0.01317	max	avg
79	98.00	39.00	58.80	15.60	74.40	8.724138	3.705882	0.01314	avg	avg
80	86.00	58.00	51.60	23.20	74.80	7.068966	5.941176	0.01327	max	avg
81	84.00	62.00	50.40	24.80	75.20	6.793103	6.411765	0.01338	max	avg
82	80.00	70.00	48.00	28.00	76.00	6.241379	7.352941	0.0136	max	avg
83	90.00	56.00	54.00	22.40	76.40	7.62069	5.705882	0.01368	max	avg
84	82.00	71.00	49.20	28.40	77.60	6.517241	7.470588	0.01401	max	avg
85	84.00	68.00	50.40	27.20	77.60	6.793103	7.117647	0.01401	max	avg
86	86.00	65.00	51.60	26.00	77.60	7.068966	6.764706	0.014	max	avg
87	92.00	65.00	55.20	26.00	81.20	7.896552	6.764706	0.01494	max	max
88	84.00	77.00	50.40	30.80	81.20	6.793103	8.176471	0.01495	max	max
89	88.00	72.00	52.80	28.80	81.60	7.344828	7.588235	0.01505	max	max
90	100.00	58.00	60.00	23.20	83.20	9	5.941176	0.01545	max	max
91	100.00	83.00	60.00	33.20	93.20	9	8.882353	0.01807	max	max
92	100.00	84.00	60.00	33.60	93.60	9	9	0.01818	max	max